Ratio Test

Let $\sum_{i=1}^{\infty} a_n$ and we define $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

Then:

i. If $L < 1$ the series is absolutely convergent

ii. If $L > 1$ the series is divergent

iii. Otherwise (i.e. $L = 0$), the test is inconclusive.

1. $L = \lim_{n\to\infty} \frac{3^{1-2n}}{n^2+1}$

2. $L = \lim_{n\to\infty} \frac{(2n)!}{5n+1}$

3. $L = \lim_{n\to\infty} \frac{(-2)^{1+3n}(n+1)}{n^2 5^{1+n}}$

4. $L = \lim_{n\to\infty} \dfrac{e^{4n}}{(n-2)!}$

5. $L = \lim_{n\to\infty} \dfrac{(-1)^{n+1}}{6n+7}$

Answers: (1) limit is $\frac{1}{9} < 1$ Abs convergence (2) limit is $\infty > 1$ diverges (3) limit is $\frac{8}{5} > 1$ diverges (4) limit is $0 \not> 1$, so converges (5) limit is 1, so the Ratio Test is inconclusive. Try the AST with $|a_n|$

Full Solutions at `https://tutorial.math.lamar.edu/Problems/CalcII/RatioTest.aspx`

Root Test

Let $\sum_{i=1}^{\infty} a_n$ and we define $L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} |a_n|^{1/n}$.

Then:

i. If $L < 1$ the series is absolutely convergent

ii. If $L > 1$ the series is divergent

iii. Otherwise (i.e. $L = 0$), the test is inconclusive.

6. $L = \lim_{n\to\infty} \left(\frac{3n+1}{4-2n}\right)^{2n}$

7. $L = \lim_{n\to\infty} \frac{n^{1-3n}}{4^{2n}}$

8. $L = \lim_{n\to\infty} \frac{(-5)^{1+2n}}{2^{5n-3}}$

Answers: (6) limit is $\frac{9}{4} > 1$ diverges (7) limit is $0 < 1$ Abs. conv. (3) limit is $\frac{25}{32} < 1$ Ab conv.

Full Solutions at `https://tutorial.math.lamar.edu/Problems/CalcII/RootTest.aspx`