AP Calculus BC Quiz 2 Name:KEY Block: Seat:

1. *(Inspired by 2004, #5, a 15 minute "No Calculator" question– feel free to use a calculator today)*

   A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{4}\left(1 - \frac{P}{18}\right).$$

   (a) (2 points) If $P(0) = 5$, what is $\lim_{t\to\infty} P(t)$? 18
   If $P(0) = 25$, what is $\lim_{t\to\infty} P(t)$?18 (1 point per answer)

   (b) (1 point) If $P(0)$ =5, for what value of P is the population growing the fastest? 9

   (c) (5 points) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{4}\left(1 - \frac{t}{18}\right).$$

   Find $Y(t)$ is $Y(0) = 5$

   Separate variables(1 point) (0/5 is no separation of variables):

$$\frac{1}{Y}dY = \frac{1}{4}\left(1 - \frac{t}{18}\right)dt = \left(\frac{1}{4} - \frac{t}{72}\right)dt$$

   Antiderivative (1 point), Constant of Integration (1 point):

$$\ln|Y| = \frac{t}{4} - \frac{t^2}{144} + C_1$$

   Uses Initial Condition (1 point) (Max is 2/5 [1-1-0-0-0] if no constant of integration)

$$Y(t) = C_2 e^{\frac{t}{4} - fract^2 144}$$

$$C_2 = 5$$

   Solve for Y(1point)

$$Y(t) = 5e^{\frac{t}{4} - fract^2 144}$$

   (d) (1 point) For the function Y found in part (c), what is the $\lim_{t\to\infty} Y(t)$?0

2. (2 points) *(Review)* Consider the curve given by $x^2 + 5y^2 = 9 + 2xy$. Show that $\frac{dy}{dx} = \frac{x-y}{x-5y}$

$$2x + 10y\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y$$

$$\frac{dy}{dx}(10y - 2x) = 2y - 2x$$

$$\frac{dy}{dx} = \frac{2y-2x}{10y-2x} = \frac{y-x}{5y-x} = \frac{x-y}{x-5y}$$