AP Calculus BC Quiz 1 KEY Block: Seat:

1. *(Inspired by 2007 form B, #5, which used steps of $\frac{1}{2}$ on a No Calculator question– feel free to use a calculator today)* Consider the differential equation $\dfrac{dy}{dx} = 2x + 3y - 1$

   (a) (2 points) Find $\dfrac{d^2y}{dx^2}$ in terms of x and y

   $$\frac{d^2y}{dx^2} = 2 + 3\frac{dy}{dx} - 0 = 2 + 3(2x + 3y - 1) = 6x + 9y - 1$$

   (b) (2 points) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = -2$. Use Euler's method, starting at $x = 0$ with a step size of 0.25 , to approximate $f(1)$. Show the work that leads to your answer.

step	x	y	Δx	$\Delta y \approx dy = f'(x,y) * dx$	New $x = x + dx$	New $y = y + dy$
0	0	-2	.25	$[2(0) + 3(-2) - 1] * .25 = -1.75$	$0 + .25 = .25$	$-2 - 1.75 = -3.75$
1	.25	-3.75	.25	$[2(.25) + 3(-3.75) - 1] * .25 = -2.9375$	$.25 + .25 = .5$	$-3.75 - 2.62625 = -6.6875$
2	.5	-6.6875	.25	$[2(.5) + 3(-6.6875) - 1] * .25 = -5.015625$	$.5 + .25 = .75$	$-6.6875- = -11.703125$
3	.75	-11.703125	.25	$[2(.75) + 3(-11.703125) - 1] * .25 = -8.65234375$	$.75 + .25 = 1$	$-11.703125 - 8.65234375 = -20.35546875$
4	1	-20.35546875				

   $f(1) \approx -20.35546875$

   (c) (1 point) Explain why you think part (b) is either an underestimate or an overestimate.

   $\dfrac{d^2y}{dx^2}(0) = 6(0) + 9(-2) - 1 < 0$, concave down, so -20.35546875 is an overestimate.

   (d) (2 points) Let $y = g(x)$ be another solution to the differential equation $\dfrac{dy}{dx} = 2x + 3y - 1$ with the initial condition $g(0) = k$, where k is a constant. Euler's method, starting at $x = 0$ with a step size of 1, gives the approximation $g(1) \approx 0$. Find the value of k.

   $g(1) \approx g(0) + g'(0) * dx$
   Since $dx = 1$ and $g'(0) = 2 * 0 + 3 * k - 1 = 3k - 1$ we have
   $g(1) \approx k + (3k - 1)(1) = 0$, Hence $k = \frac{1}{4}$

Bonus (2 points): Find the values of the constants m, b and r for which $y = mx + b + e^{rx}$ is a solution to the differential equation $\dfrac{dy}{dx} = 2x + 3y - 1$

If $y = mx + b + e^{rx}$ is a solution, then $m + re^{rx} = 2x + 3(mx + b + e^{rx}) - 1$
If $r \neq 0$, then $m = 3b - 1$, $r = 3$, $0 = 2 + 3m$ so $m = -\frac{2}{3}$, $r = 3$ and $b = \frac{1}{9}$
If $r = 0$, then $m = 3b + 2$, $r = 0$, $0 = 2 + 3m$ so $m = -\frac{2}{3}$, $r = 0$, $b = \frac{4}{9}$