Legrange WS3 Name: Block: Seat:

1. Let f be a function that has derivatives of all orders for all real numbers x. Assume that $f(5) = 6$, $f'(5) = 8$, $f''(5) = 30$, $f'''(5) = 48$, and $\left|f^{(4)}(x)\right| \leq 45$,
   (a) Find the third-degree Taylor polynomial about $x = 5$ for $f(X)$.

   (b) Use your answer to part (a) to estimate the value of $f(5.2)$. What is the maximum possible error in making this estimate? Give three decimal places.

(c) Use your answer to (b) to find an interval $[a, b]$ such that $a \leq f(5.2) \leq b$. Give three decimal places.

(d) Could $f(5.2)$ equal 8.254? Show why or why not.

2. Let f be the function given by $f(x) = \cos\left(2x + \frac{\pi}{6}\right)$ and let $P_3(x)$ be the third-degree Taylor polynomial for f about $x = 0$.

   (a) Find $P_3(x)$

   (b) Use the Lagrange error bound to show that $\left| f\left(\frac{1}{10}\right) - P\left(\frac{1}{10}\right) \right| < \frac{1}{12,000}$

3. (Calculator OK) Let f be a function that has derivatives of all orders. Assume $f(3) = 1$, $f'(3) = \frac{1}{2}$, $f''(3) = -\frac{1}{4}$, $f'''(3) = \frac{3}{8}$ and the graph of $f^{(4)}(x)$ on $[3, 4]$ is increasing, where $f^{(4)}(3) = 2$ and $f^{(4)}(4) = 6$

(a) Find the third-degree Taylor polynomial $P_3(x)$ about $x = 3$ for the function f.

(b) Use your answer to part (a) to estimate the value of $f(3.7)$

(c) Show that $|f(3.7) - P_3(3.7)| < 0.08$

(d) Could equal 1.283? Show why or why not.

4. (Calculator OK) Find the maximum error incurred by approximating the sum of the series

$$1 - \frac{1}{2!} + \frac{2}{3!} - \frac{3}{4!} + \frac{4}{5!} - \cdots + \frac{(-1)^{n+1}(n-1)}{n!} + \ldots$$

by the sum of the first five terms. Justify your answer.

5. Let f be the function given by $f(x) = \cos\left(3x + \frac{\pi}{6}\right)$ and let $P_4(x)$ be the fourth-degree Taylor polynomial for f about $x = 0$.

(a) Find $P_4(x)$

(b) Use the Lagrange error bound to show that $\left|f\left(\frac{1}{6}\right) - P\left(\frac{1}{6}\right)\right| < \frac{1}{3,000}$. Justify your answer.

6. Use series to find an estimate for $\int_0^1 e^{-x^2}\,dx$ so that the error is less that $\frac{1}{200}$. Justify your answer.

7. (Calculator OK) Suppose a function f is approximated with a fourth-degree Taylor polynomial about $x = 1$. If the maximum value of the fifth derivative between $x = 1$ and $x = 3$ is 0.01 , that is, $\left|f^{(5)}(x) < 0.01\right|$, find the maximum error incurred using this approximation to compute $f(3)$.