

69. $y' = x + y$, $y(0) = 2$, $n = 10$, $h = 0.1$

$$y_1 = y_0 + hF(x_0, y_0) = 2 + (0.1)(0 + 2) = 2.2$$

$$y_2 = y_1 + hF(x_1, y_1) = 2.2 + (0.1)(0.1 + 2.2) = 2.43, \text{ etc.}$$

n	0	1	2	3	4	5	6	7	8	9	10
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y_n	2	2.2	2.43	2.693	2.992	3.332	3.715	4.146	4.631	5.174	5.781

70. $y' = x + y$, $y(0) = 2$, $n = 20$, $h = 0.05$

$$y_1 = y_0 + hF(x_0, y_0) = 2 + (0.05)(0 + 2) = 2.1$$

$$y_2 = y_1 + hF(x_1, y_1) = 2.1 + (0.05)(0.05 + 2.1) = 2.2075, \text{ etc.}$$

The table shows the values for $n = 0, 2, 4, \dots, 20$.

n	0	2	4	6	8	10	12	14	16	18	20
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y_n	2	2.208	2.447	2.720	3.032	3.387	3.788	4.240	4.749	5.320	5.960

71. $y' = 3x - 2y$, $y(0) = 3$, $n = 10$, $h = 0.05$

$$y_1 = y_0 + hF(x_0, y_0) = 3 + (0.05)(3(0) - 2(3)) = 2.7$$

$$y_2 = y_1 + hF(x_1, y_1) = 2.7 + (0.05)(3(0.05) - 2(2.7)) = 2.4375, \text{ etc.}$$

n	0	1	2	3	4	5	6	7	8	9	10
x_n	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
y_n	3	2.7	2.438	2.209	2.010	1.839	1.693	1.569	1.464	1.378	1.308

72. $y' = 0.5x(3 - y)$, $y(0) = 1$, $n = 5$, $h = 0.4$

$$y_1 = y_0 + hF(x_0, y_0) = 1 + (0.4)(0.5(0)(3 - 1)) = 1$$

$$y_2 = y_1 + hF(x_1, y_1) = 1 + (0.4)(0.5(0.4)(3 - 1)) = 1.16, \text{ etc.}$$

n	0	1	2	3	4	5
x_n	0	0.4	0.8	1.2	1.6	2.0
y_n	1	1	1.16	1.454	1.825	2.201

73. $y' = e^{xy}$, $y(0) = 1$, $n = 10$, $h = 0.1$

$$y_1 = y_0 + hF(x_0, y_0) = 1 + (0.1)e^{0(1)} = 1.1$$

$$y_2 = y_1 + hF(x_1, y_1) = 1.1 + (0.1)e^{(0.1)(1.1)} \approx 1.2116, \text{ etc.}$$

n	0	1	2	3	4	5	6	7	8	9	10
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y_n	1	1.1	1.212	1.339	1.488	1.670	1.900	2.213	2.684	3.540	5.958

74. $y' = \cos x + \sin y$, $y(0) = 5$, $n = 10$, $h = 0.1$

$$y_1 = y_0 + hF(x_0, y_0) = 5 + (0.1)(\cos 0 + \sin 5) \approx 5.0041$$

$$y_2 = y_1 + hF(x_1, y_1) = 5.0041 + (0.1)(\cos(0.1) + \sin(5.0041)) \approx 5.0078, \text{ etc.}$$

n	0	1	2	3	4	5	6	7	8	9	10
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y_n	5	5.004	5.008	5.010	5.010	5.007	4.999	4.985	4.965	4.938	4.903

75. $\frac{dy}{dx} = y$, $y = 3e^x$, $y(0) = 3$

x	0	0.2	0.4	0.6	0.8	1.0
$y(x)$ (exact)	3	3.6642	4.4755	5.4664	6.6766	8.1548
$y(x)$ ($h = 0.2$)	3	3.6000	4.3200	5.1840	6.2208	7.4650
$y(x)$ ($h = 0.1$)	3	3.6300	4.3923	5.3147	6.4308	7.7812

76. $\frac{dy}{dx} = \frac{2x}{y}$, $y = \sqrt{2x^2 + 4}$, $y(0) = 2$

x	0	0.2	0.4	0.6	0.8	1.0
$y(x)$ (exact)	2	2.0199	2.0785	2.1726	2.2978	2.4495
$y(x)$ ($h = 0.2$)	2	2.000	2.0400	2.1184	2.2317	2.3751
$y(x)$ ($h = 0.1$)	2	2.0100	2.0595	2.1460	2.2655	2.4131

77. $\frac{dy}{dx} = y + \cos x$, $y = \frac{1}{2}(\sin x - \cos x + e^x)$, $y(0) = 0$

78. As h increases (from 0.1 to 0.2), the error increases.

x	0	0.2	0.4	0.6	0.8	1.0
$y(x)$ (exact)	0	0.2200	0.4801	0.7807	0.1231	0.5097
$y(x)$ ($h = 0.2$)	0	0.2000	0.4360	0.7074	0.0140	0.3561
$y(x)$ ($h = 0.1$)	0	0.2095	0.4568	0.7418	0.0649	0.4273

79. $\frac{dy}{dt} = -\frac{1}{2}(y - 72)$, $y(0) = 140$, $h = 0.1$

80. $\frac{dy}{dt} = -\frac{1}{2}(y - 72)$, $y(0) = 140$, $h = 0.05$

(a)

t	0	1	2	3
Euler	140	112.7	96.4	86.6

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Euler	140	112.98	96.7	86.9

(b) $y = 72 + 68e^{-t/2}$ exact

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t	0	1	2	3
Exact	140	113.24	97.016	87.173

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Exact	140	113.24	97.016	87.173

The approximations are better using $h = 0.05$.