NO CALCULATORS permitted this section (60min)

NO calculator Section (Must be turned in before Calculators): 2 Free Response questions (15 minutes each = 30 min); 14 Multiple choice questions (2 minutes a question= 28 min).

Calculator Section: 1 Free Response question (15 minutes); 5 Multiple choice questions (15 minutes). 100 points for the Multiple choice, 100 points for the free response, the 2 quarter exams are worth 15% of your grade. For choice (f) fill the (a) and (b) bubbles of your scantron.

NC-I. Let $a_n = \dfrac{1}{n \ln n}$ for $n \geq 3$.

(a) Let f be the function given by $f(x) = \dfrac{1}{x \ln x}$. For $x \geq 3$, f is continuous, decreasing, and positive. Use either the integral test or the comparison test to show that $\displaystyle\sum_{n=3}^{\infty} a_n$ diverges.

(b) Consider the infinite series $\displaystyle\sum_{n=3}^{\infty} (-1)^{n+1} a_n = \frac{1}{3\ln 3} - \frac{1}{4\ln 4} + \frac{1}{5\ln 5} - \cdots$ Identify properties of this series that guarentee the series converges. Explain why the sum of this eries is less than $\dfrac{1}{3}$.

(c) Find the interval of convergence of the power series $\sum_{n=3}^{\infty} \frac{(x-2)^{n+1}}{n \ln n}$

NC-II. The function f satisfies the equation

$$f'(x) = f(x) + x + 1$$

and $f(0) = 2$. The Taylor series for f about $x = 0$ converges to $f(x)$ for all x.

(a) Write an equation for the line tangent to the curve $y = f(x)$ at the point where $x = 0$.

(b) Find $f''(0)$ and find the second-degree Taylor polynomial for f about $x = 0$.

(c) Find the fourth degree Taylor polynomial for f about $x = 0$.

(d) Find $f^{(n)}(0)$, the nth derivative of f at $x = 0$, for $n \geq 2$. Use the Taylor series for f about $x = 0$ and the Taylor series for e^x about $x = 0$ to find a polynomial expression for $f(x) = 4e^x$

For choice (f) fill the (a) and (b) bubbles of your scantron.

1. $\lim_{x\to 0} \dfrac{x^2}{1-\cos x}$ is

   (a) -2
   (b) 0
   (c) 1
   (d) 2
   (e) nonexistant
   (f) None of these

2. $\displaystyle\int \frac{1}{x^2-7x+10}\,dx$

   (a) $\ln|(x-2)(x-5)|+C$
   (b) $\dfrac{1}{3}\ln|(x-2)(x-5)|+C$
   (c) $\dfrac{1}{3}\ln\left|\dfrac{2x-7}{(x-2)(x-5)}\right|+C$
   (d) $\dfrac{1}{3}\ln\left|\dfrac{x-2}{x-5}\right|+C$
   (e) $\dfrac{1}{3}\ln\left|\dfrac{x-5}{x-2}\right|+C$
   (f) None of these

3. The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = (-1)^{n+1}$ for $n \geq 1$. What is the sum of the series?
   (a) -1
   (b) 0
   (c) $\frac{1}{2}$
   (d) 1
   (e) The series diverges.
   (f) None of these

4. What is the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$?
   (a) $\frac{-2}{e^2 - 2e}$
   (b) $\frac{-2}{e^2 + 2e}$
   (c) $\frac{-2}{e + 2}$
   (d) $\frac{e}{e + 2}$
   (e) The series diverges
   (f) None of these

5. Let $P(x) = 3 - 3x^2 + 6x^4$ be the fourth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f^{(4)}(0)$?
   (a) 0
   (b) $\frac{1}{4}$
   (c) 6
   (d) 24
   (e) 144
   (f) None of these

6. Which of the following is the Maclaurin series for e^{3x}?
   (a) $1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$
   (b) $3 + 9x + \frac{27x^2}{2} + \frac{81x^3}{3!} + \frac{243x^4}{4!} + \cdots$
   (c) $1 - 3x + \frac{9x^2}{2} - \frac{27x^3}{3!} + \frac{81x^4}{4!} - \cdots$
   (d) $1 + 3x + \frac{3x^2}{2} + \frac{3x^3}{3!} + \frac{3x^4}{4!} + \cdots$
   (e) $1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{3!} + \frac{81x^4}{4!} + \cdots$
   (f) None of these

x	1	3	5	7
$f(x)$	4	6	7	5
$f'(x)$	2	1	0	-1

7. The table above gives selected values for a differentiable function f and its first derivative. Using a left Riemann sum with three subintervals of equal length, which of the follwoing is an approximation of the length of the graph of f on the interval $[1, 7]$?
   (a) 6
   (b) 34
   (c) $2\sqrt{3} + 2\sqrt{2} + 2$
   (d) $2\sqrt{5} + 2\sqrt{2} + 2$
   (e) $2\sqrt{5} + 4\sqrt{2} + 2$
   (f) None of these

8. What is the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 2^n}$
   (a) $1 < x < 5$
   (b) $1 \leq x < 5$
   (c) $1 \leq x \leq 5$
   (d) $2 < x < 4$
   (e) $2 \leq x \leq 4$
   (f) None of these

9. Which of the following series converge?

I. $1 + (-1) + 1 + \cdots + (-1)^{n-1} + \cdots$

II. $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} + \cdots$

III. $1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^{n-1}} + \cdots$

(a) I only
(b) II only
(c) III only
(d) II and III only
(e) I, II, and III
(f) None of these

10. What is the coefficient of x^2 in the Taylor series for $\sin^2 x$ about $x = 0$?

(a) -2
(b) -1
(c) 0
(d) 1
(e) 2
(f) None of these

11. If $\lim_{h \to 0} \frac{\arcsin(a+h) - \arcsin(a)}{h} = 2$, which of the following could be the value of a?

(a) $\frac{\sqrt{2}}{2}$
(b) $\frac{\sqrt{3}}{2}$
(c) $\sqrt{3}$
(d) $\frac{1}{2}$
(e) 2
(f) None of these

12. What is the value of $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{5^n}$

(a) $-\frac{15}{8}$

(b) $-\frac{9}{8}$

(c) $-\frac{3}{8}$

(d) $\frac{9}{8}$

(e) $\frac{15}{8}$

(f) None of these

13. A population y changes at a rate modeled by the differential equation $\frac{dy}{dx} = 0.2y(1000 - y)$, where t is measured in years. What are all the values of y for which the population is increasing at a decreasing rate?

(a) 500 only

(b) $0 < y < 500$ only

(c) $500 < y < 1000$ only

(d) $0 < y < 1000$

(e) $y > 1000$

(f) None of these

14. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(x + \frac{3}{2}\right)^n$ converges?

(a) $-\frac{5}{2} < x < -\frac{1}{2}$

(b) $-\frac{5}{2} < x \leq -\frac{1}{2}$

(c) $-\frac{5}{2} \leq x < -\frac{1}{2}$

(d) $-\frac{1}{2} < x < \frac{1}{2}$

(e) $x \leq -\frac{1}{2}$

(f) None of these

Calculators may be used for the following Questions

x	$f'(x)$
1	0.2
1.5	0.5
2.0	0.9

15. The table above gives values of f', the derivative of a function f. If $f(1) = 4$, what is the approximation to $f(2)$ obtained by Euler's method with a step size of 0.5?
    (a) 2.35
    (b) 3.65
    (c) 4.35
    (d) 4.70
    (e) 4.80
    (f) None of these

16. If $0 < b_n < a_n$ for $n \geq 1$, which of the following must be true?
    (a) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} b_n$ converges.
    (b) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} b_n = 0$.
    (c) If $\sum_{n=1}^{\infty} b_n$ diverges, then $\lim_{n\to\infty} a_n = 0$.
    (d) If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.
    (e) If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
    (f) None of these

x	2	4
$f(x)$	7	13
$g(x)$	2	9
$g'(x)$	1	7
$g''(x)$	5	8

17. The table above gives selected values of twice differentiable functions f and g, as well as the first two derivatives of g. If $f'(x) = 3$ for all values of x, what is the value of $\int_2^4 f(x)g''(x)\,dx$?
    (a) 63
    (b) 69
    (c) 78
    (d) 84
    (e) 103
    (f) None of these

18. $\lim_{x\to 0}(1+2x)^{\csc x}$
    (a) 0
    (b) 1
    (c) 2
    (d) e
    (e) e^2
    (f) None of these

19. If $f(x) = \sum_{k=1}^{\infty}(\sin^2 x)^k$, then $f(1)$ is
    (a) 0.369
    (b) 0.585
    (c) 2.400
    (d) 2.426
    (e) 3.426
    (f) None of these

20. The function f has a Taylor series about $x = 2$ that converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 2$ is given by $f^{(n)}(2) = \dfrac{(n+1)!}{3^n}$ for $n \geq 1$, and $f(2) = 1$.

(a) Write the first four terms and the general term of the Taylor series for f about $x = 2$.

(b) Find the radius of convergence for the Taylor series for f about $x = 2$. Show the work that led to your answer.

(c) Let g be a function satisfying $g(2) = 3$ and $g'(x) = f(x)$ for all x. Write the first four terms and the general term of the Taylor series for g about $x = 2$.

(d) Does the Taylor series for g as defined in part (c) converge at $x = -2$? Give a reason for your answer