Free Response-Calculators Permitted

C-I. Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure.

(a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k.

(b) The region R is rotated about the x-axis to form a solid. Find the surface area, S, of the solid in terms of k.*Hint:* The surface area is 2π times an integral that looks a lot like the arc length integral, the only difference is that it should be multiplied by the changing radius.

(c) The surface area S, found in part (b) changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dS}{dt}$ when $k = \frac{1}{2}$

15.

$$\lim_{x\to 0^+} \frac{1+\sin x}{x} =$$

(a) 0
(b) 1
(c) 2
(d) π
(e) ∞
(f) None of these

16. If $\dfrac{dy}{dx} = 2x + y$ and $y(1) = 3$, use Euler's method with step size 0.2 to approximate $y(1.4)$

(a) 3.40
(b) 4.00
(c) 5.20
(d) 5.28
(e) 6.40
(f) None of these

17. If the length of the graph of $y = f(x)$ on the interval $[0, 1]$is

$$\int_0^1 \sqrt{1 + 36e^{6x}}\, dx,$$

then $f(x) =$

(a) $2e^{3x}$
(b) $6e^{3x}$
(c) $12e^{3x}$
(d) e^{6x}
(e) $6e^{6x}$
(f) None of these

18.

$$\int x^2 \ln x \; dx =$$

(a) $\dfrac{x^2}{2} + C$

(b) $\dfrac{x^3}{3} \ln x - \dfrac{x^3}{3} + C$

(c) $\dfrac{x^3}{3} \ln x - \dfrac{x^3}{9} + C$

(d) $\dfrac{x^3}{3} \ln x - \dfrac{x^4}{12} + C$

(e) $\dfrac{x^3}{3} \ln x + \dfrac{x^4}{12} + C$

(f) None of these

19.

$$\int x^3 \sin(x^2) \; dx =$$

(a) $2x - \dfrac{1}{2} \cos(x^2) + C$

(b) $\dfrac{1}{2} \left(x^2 \cos(x^2) - \sin(x^2) \right) + C$

(c) $-\dfrac{1}{2} \left(x^2 \cos(x^2) - \sin(x^2) \right) + C$

(d) $x^2 \cos(x^2) - \sin(x^2) + C$

(e) $x^2 \cos(x^2) + \sin(x^2) + C$

(f) None of these