## **AP Calculus – Final Review Sheet**

When you see the words	This is what you think of doing
1. Find the zeros	
2. Find equation of the line tangent to $f(x)$ at $(a,b)$	
3. Find equation of the line normal to $f(x)$ at $(a,b)$	
4. Show that $f(x)$ is even	
5. Show that $f(x)$ is odd	
6. Find the interval where $f(x)$ is increasing	
7. Find interval where the slope of $f(x)$ is increasing	
8. Find the minimum value of a function	
9. Find the minimum slope of a function	
10. Find critical values	
11. Find inflection points	
12. Show that $\lim_{x \to a} f(x)$ exists	
13. Show that $f(x)$ is continuous	
14. Find vertical asymptotes of $f(x)$	
15. Find horizontal asymptotes of $f(x)$	

16. Find the average rate of change of $f(x)$ on $[a,b]$	
17. Find instantaneous rate of change of $f(x)$ at $a$	
18. Find the average value of $f(x)$ on $[a,b]$	
19. Find the absolute maximum of $f(x)$ on $[a,b]$	
20. Show that a piecewise function is differentiable at the point $a$ where the function rule splits	
21. Given $s(t)$ (position function), find $v(t)$	
22. Given $v(t)$ , find how far a particle travels on $[a,b]$	
23. Find the average velocity of a particle on $[a,b]$	
24. Given $v(t)$ , determine if a particle is speeding up at $t = k$	
25. Given $v(t)$ and $s(0)$ , find $s(t)$	
26. Show that Rolle's Theorem holds on $[a,b]$	
27. Show that Mean Value Theorem holds on $[a,b]$	
28. Find domain of $f(x)$	
29. Find range of $f(x)$ on $[a,b]$	
30. Find range of $f(x)$ on $(-\infty,\infty)$	
31. Find $f'(x)$ by definition	
32. Find derivative of inverse to $f(x)$ at $x = a$	
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33. $y$ is increasing proportionally to $y$	
<i>y</i> is mercusing proportionally to <i>y</i>	
34. Find the line $x = c$ that divides the area under	
f(x) on $[a,b]$ to two equal areas	
35. $\frac{d}{dx}\int_{a}^{x}f(t)dt =$	
36. $\frac{d}{dx} \int_{a}^{b} f(t) dt$	
37. The rate of change of population is	
38. The line $y = mx + b$ is tangent to $f(x)$ at $(a,b)$	
39. Find area using left Riemann sums	
40. Find area using right Riemann sums	
41. Find area using midpoint rectangles	
42. Find area using trapezoids	
42. Solve the differential equation	
43. Solve the differential equation	
44. Meaning of $\int_{0}^{x} f(t) dt$	
44. Meaning of <i>f</i> ( <i>t</i> ) <i>at</i>	
45. Given a base, cross sections perpendicular to the	
<i>x</i> -axis are squares	
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46. Find where the tangent line to $f(x)$ is horizontal	
47. Find where the tangent line to $f(x)$ is vertical	
48. Find the minimum acceleration given $v(t)$	
49. Approximate the value of $f(0.1)$ by using the	
tangent line to f at $x = 0$	

50. Given the value of $f(a)$ and the fact that the anti- derivative of $f$ is $F$ , find $F(b)$	
51. Find the derivative of $f(g(x))$	
52. Given $\int_{a}^{b} f(x) dx$ , find $\int_{a}^{b} [f(x)+k] dx$	
53. Given a picture of $f'(x)$ , find where $f(x)$ is increasing	
54. Given $v(t)$ and $s(0)$ , find the greatest distance from the origin of a particle on $[a,b]$	
55. Given a water tank with g gallons initially being filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[t_1, t_2]$ , find a) the amount of water in the tank at m minutes	
56. b) the rate the water amount is changing at <i>m</i>	
57. c) the time when the water is at a minimum	
58. Given a chart of x and $f(x)$ on selected values between a and b, estimate $f'(c)$ where c is between a and b.	
59. Given $\frac{dy}{dx}$ , draw a slope field	
60. Find the area between curves $f(x), g(x)$ on $[a,b]$	
61. Find the volume if the area between $f(x)g(x)$ is rotated about the <i>x</i> -axis	

## **BC Problems**

62. Find $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ if $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} g(x) = 0$	
63. Find $\int_0^\infty f(x) dx$	
64. $\frac{dP}{dt} = \frac{k}{M}P(M-P)$ or $\frac{dP}{dt} = kP\left(1-\frac{P}{M}\right)$	
65. Find $\int \frac{dx}{x^2 + ax + b}$ where $x^2 + ax + b$ factors	
<ul> <li>66. The position vector of a particle moving in the plane is r(t) = (x(t), y(t))</li> <li>a) Find the velocity.</li> </ul>	
67. b) Find the acceleration.	
68. c) Find the speed.	
69. a) Given the velocity vector $v(t) = \langle x(t), y(t) \rangle$ and position at time 0, find the position vector.	
70. b) When does the particle stop?	
71. c) Find the slope of the tangent line to the vector at $t_1$ .	
72. Find the area inside the polar curve $r = f(\theta)$ .	
73. Find the slope of the tangent line to the polar curve $r = f(\theta)$ .	
74. Use Euler's method to approximate $f(1.2)$ given	
$\frac{dy}{dx}$ , $(x_0, y_0) = (1,1)$ , and $\Delta x = 0.1$	
75. Is the Euler's approximation an underestimate or an overestimate?	
76. Find $\int x^n e^{ax} dx$ where <i>a</i> , <i>n</i> are integers	
77. Write a series for $x^n \cos x$ where <i>n</i> is an integer	

78. Write a series for $\ln(1+x)$ centered at $x=0$ .	
79. $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if	
80. If $f(x) = 2 + 6x + 18x^2 + 54x^3 + \dots$ , find $f\left(-\frac{1}{2}\right)$	
81. Find the interval of convergence of a series.	
82. Let $S_4$ be the sum of the first 4 terms of an alternating series for $f(x)$ . Approximate $ f(x) - S_4 $	
83. Suppose $f^{(n)}(x) = \frac{(n+1) n!}{2^n}$ . Write the first four terms and the general term of a series for $f(x)$ centered at $x = c$	
84. Given a Taylor series, find the Lagrange form of the remainder for the $4^{th}$ term.	
85. $f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$	
86. $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \dots$	
87. $f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + \frac{(-1)^n x^{2n}}{(2n)!} + \dots$	
88. Find $\int (\sin x)^m (\cos x)^n dx$ where <i>m</i> and <i>n</i> are integers	
89. Given $x = f(t), y = g(t)$ , find $\frac{dy}{dx}$	
90. Given $x = f(t), y = g(t)$ , find $\frac{d^2 y}{dx^2}$	
91. Given $f(x)$ , find arc length on $[a,b]$	
92. $x = f(t), y = g(t)$ , find arc length on $[t_1, t_2]$	
93. Find horizontal tangents to a polar curve $r = f(\theta)$	
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94. Find vertical tangents to a polar curve $r = f(\theta)$	
95. Find the volume when the area between $y = f(x), x = 0, y = 0$ is rotated about the <i>y</i> -axis.	
96. Given a set of points, estimate the volume under the curve using Simpson's rule.	

## AP Calculus – Final Review Sheet

1. Find the zerosSet function = 0, factor or use quadratic equation if quadratic, graph to find zeros on calculator2. Find equation of the line tangent to $f(x)$ on $[a,b]$ Take derivative - $f'(a) = m$ and use $y - y_1 = m(x - x_1)$ 3. Find equation of the line normal to $f(x)$ on $[a,b]$ Same as above but $m = \frac{-1}{f'(a)}$ 4. Show that $f(x)$ is evenShow that $f(-x) = f(x)$ - symmetric to y-axis5. Show that $f(x)$ is oddShow that $f(-x) = -f(x)$ - symmetric to origin6. Find the interval where $f(x)$ is increasingFind $f'(x)$ , set both numerator and denominator to zero to find critical points, make sign chart of $f'(x)$ and determine where it is positive.7. Find interval where the slope of $f(x)$ is increasingFind the derivative of $f'(x) = f''(x)$ , set both numerator and denominator to zero to find critical points, make sign chart of $f''(x)$ and determine where it is positive.8. Find the minimum value of a functionMake a sign chart of $f'(x)$ , find all relative minimum
2. Find equation of the line tangent to $f(x)$ on $[a,b]$ Take derivative - $f'(a) = m$ and use $y - y_1 = m(x - x_1)$ 3. Find equation of the line normal to $f(x)$ on $[a,b]$ Same as above but $m = \frac{-1}{f'(a)}$ 4. Show that $f(x)$ is evenShow that $f(-x) = f(x)$ - symmetric to y-axis5. Show that $f(x)$ is oddShow that $f(-x) = -f(x)$ - symmetric to origin6. Find the interval where $f(x)$ is increasingFind $f'(x)$ , set both numerator and denominator to zero to find critical points, make sign chart of $f'(x)$ and determine where it is positive.7. Find interval where the slope of $f(x)$ is increasingFind the derivative of $f'(x) = f''(x)$ , set both numerator and denominator to zero to find critical points, make sign chart of $f''(x)$ and determine where it is positive.
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5. Show that $f(x)$ is odd 6. Find the interval where $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find interval where the slope of $f(x)$ is increasing 7. Find the derivative of $f'(x) = f''(x)$ , set both 7. Find interval where the slope of $f(x)$ is increasing 7. Find the derivative of $f'(x) = f''(x)$ , set both 7. Find interval where the slope of $f(x)$ is increasing 7. Find the derivative of $f'(x)$ and determine where 8. Find the derivative of $f''(x)$ and determine where 8. Find the derivative of
<ul> <li>6. Find the interval where f(x) is increasing</li> <li>7. Find interval where the slope of f(x) is increasing</li> <li>7. Find interval where the slope of f(x) is increasing</li> <li>7. Find interval where the slope of f(x) is increasing</li> <li>7. Find the derivative of f'(x) = f''(x), set both numerator and denominator to zero to find critical points, make sign chart of f''(x) and determine where it is positive.</li> </ul>
zero to find critical points, make sign chart of $f'(x)$ and determine where it is positive.7. Find interval where the slope of $f(x)$ is increasingFind the derivative of $f'(x) = f''(x)$ , set both numerator and denominator to zero to find critical points, make sign chart of $f''(x)$ and determine wher it is positive.
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it is positive.
8. Find the minimum value of a function Make a sign chart of $f'(x)$ find all relative minimum
8. The die minimum value of a function while a sign chart of $f(x)$ , find an relative minimum
and plug those values back into $f(x)$ and choose the
smallest.
9. Find the minimum slope of a function Make a sign chart of the derivative of $f'(x) = f''(x)$ ,
find all relative minimums and plug those values back
into $f'(x)$ and choose the smallest.
10. Find critical values Express $f'(x)$ as a fraction and set both numerator
and denominator equal to zero.
11. Find inflection points Express $f''(x)$ as a fraction and set both numerator
and denominator equal to zero. Make sign chart of
f''(x) to find where $f''(x)$ changes sign. (+ to - or -
to +)
12. Show that $\lim_{x \to a^-} f(x)$ exists Show that $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$
13. Show that $f(x)$ is continuous Show that 1) $\lim_{x \to a} f(x)$ exists $(\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x))$
2) $f(a)$ exists
3) $\lim_{x \to a} f(x) = f(a)$
14. Find vertical asymptotes of $f(x)$ Do all factor/cancel of $f(x)$ and set denominator = 0
15. Find horizontal asymptotes of $f(x)$ Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to \infty} f(x)$
16. Find the average rate of change of $f(x)$ on $[a,b]$ Find $\frac{f(b)-f(a)}{b-a}$
17. Find instantaneous rate of change of $f(x)$ at $a$ Find $f'(a)$

19 Find the every set $f(x) = \begin{bmatrix} x \end{bmatrix}$	b
18. Find the average value of $f(x)$ on $[a,b]$	Find $\frac{\int_{a}^{b} f(x) dx}{b - a}$
	Find $\frac{a}{b-a}$
19. Find the absolute maximum of $f(x)$ on $[a,b]$	Make a sign chart of $f'(x)$ , find all relative
	maximums and plug those values back into $f(x)$ as
	well as finding $f(a)$ and $f(b)$ and choose the largest.
20. Show that a piecewise function is differentiable	First, be sure that the function is continuous at $x = a$ .
at the point <i>a</i> where the function rule splits	Take the derivative of each piece and show that
	$\lim_{x \to a^-} f'(x) = \lim_{x \to a^+} f'(x)$
21. Given $s(t)$ (position function), find $v(t)$	Find $v(t) = s'(t)$
22. Given $v(t)$ , find how far a particle travels on $[a,b]$	Find $\int_{a}^{b}  v(t)  dt$
23. Find the average velocity of a particle on $[a,b]$	
	Find $\frac{\int v(t) dt}{b-a} = \frac{s(b) - s(a)}{b-a}$
24. Given $v(t)$ , determine if a particle is speeding up	Find $v(k)$ and $a(k)$ . Multiply their signs. If both
at $t = k$	positive, the particle is speeding up, if different signs,
	then the particle is slowing down.
25. Given $v(t)$ and $s(0)$ , find $s(t)$	$s(t) = \int v(t) dt + C$ Plug in $t = 0$ to find C
26. Show that Rolle's Theorem holds on $[a,b]$	Show that <i>f</i> is continuous and differentiable on the
	interval. If $f(a) = f(b)$ , then find some c in $[a,b]$
	such that $f'(c) = 0$ .
27. Show that Mean Value Theorem holds on $[a,b]$	Show that $f$ is continuous and differentiable on the
	interval. Then find some c such that $f(t) = f(t)$
	$f'(c) = \frac{f(b) - f(a)}{b - a}.$
29. Find domain of $f(x)$	b-a
28. Find domain of $f(x)$	Assume domain is $(-\infty,\infty)$ . Restrictable domains:
	denominators $\neq 0$ , square roots of only non negative numbers log or ln of only positive numbers
29. Find range of $f(x)$ on $[a,b]$	numbers, log or ln of only positive numbers. Use max/min techniques to rind relative max/mins.
	Then examine $f(a), f(b)$
30. Find range of $f(x)$ on $(-\infty,\infty)$	Use max/min techniques to rind relative max/mins.
	Then examine $\lim_{x \to \pm \infty} f(x)$ .
31. Find $f'(x)$ by definition	$\frac{x \to \pm \infty^{-1} \setminus f}{f(x+h) - f(x)}$
51. Find J (x) by definition	$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \text{ or}$ $f'(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$
	$ \begin{pmatrix} \ddots & h \\ & & \end{pmatrix} \begin{pmatrix} & h \\ & & \end{pmatrix} \end{pmatrix} \begin{pmatrix} & h \\ & & \end{pmatrix} \begin{pmatrix} & h \\ & & \end{pmatrix} \begin{pmatrix} & h \\ & & \end{pmatrix} \end{pmatrix} $
	$f'(x) = \lim \frac{f(x) - f(a)}{a}$
22 Find derivative of inverse to (()) at a	
32. Find derivative of inverse to $f(x)$ at $x = a$	Interchange x with y. Solve for $\frac{dy}{dx}$ implicitly (in terms
	of $y$ ). Plug your $x$ value into the inverse relation and
	solve for y. Finally, plug that y into your $\frac{dy}{dx}$ .

33. <i>y</i> is increasing proportionally to $y$	$\frac{dy}{dt} = ky$ translating to $y = Ce^{kt}$
34. Find the line $x = c$ that divides the area under $f(x)$ on $[a,b]$ to two equal areas	$\int_{a}^{c} f(x) dx = \int_{c}^{b} f(x) dx$
$35.  \frac{d}{dx} \int_{a}^{x} f(t) dt =$	$2^{nd}$ FTC: Answer is $f(x)$
$36. \ \frac{d}{dx} \int_{a}^{u} f(t) dt$	2 <sup>nd</sup> FTC: Answer is $f(u)\frac{du}{dx}$
37. The rate of change of population is	$\frac{dP}{dt} = \dots$
38. The line $y = mx + b$ is tangent to $f(x)$ at $(x_1, y_1)$	Two relationships are true. The two functions share the same slope $(m = f'(x))$ and share the same y value at $x_1$ .
39. Find area using left Riemann sums	$A = base[x_0 + x_1 + x_2 + + x_{n-1}]$
40. Find area using right Riemann sums	$A = base[x_1 + x_2 + x_3 + + x_n]$
41. Find area using midpoint rectangles	Typically done with a table of values. Be sure to use only values that are given. If you are given 6 sets of points, you can only do 3 midpoint rectangles.
42. Find area using trapezoids	$A = \frac{base}{2} [x_0 + 2x_1 + 2x_2 + + 2x_{n-1} + x_n]$ This formula only works when the base is the same. If not, you have to do individual trapezoids.
43. Solve the differential equation	Separate the variables $-x$ on one side, y on the other. The dx and dy must all be upstairs.
44. Meaning of $\int_{a}^{x} f(t) dt$	The accumulation function – accumulated area under the function $f(x)$ starting at some constant <i>a</i> and ending at <i>x</i> .
45. Given a base, cross sections perpendicular to the <i>x</i> -axis are squares	The area between the curves typically is the base of your square. So the volume is $\int_{a}^{b} (base^{2}) dx$
46. Find where the tangent line to $f(x)$ is horizontal	Write $f'(x)$ as a fraction. Set the numerator equal to zero.
47. Find where the tangent line to $f(x)$ is vertical	Write $f'(x)$ as a fraction. Set the denominator equal to zero.
48. Find the minimum acceleration given $v(t)$	First find the acceleration $a(t) = v'(t)$ . Then minimize the acceleration by examining $a'(t)$ .
49. Approximate the value of $f(0.1)$ by using the tangent line to $f$ at $x = 0$	Find the equation of the tangent line to <i>f</i> using $y - y_1 = m(x - x_1)$ where $m = f'(0)$ and the point is $(0, f(0))$ . Then plug in 0.1 into this line being sure to use an approximate ( $\approx$ )sign.

50. Given the value of $F(a)$ and the fact that the anti- derivative of $f$ is $F$ , find $F(b)$ 1	Usually, this problem contains an antiderivative you cannot take. Utilize the fact that if $F(x)$ is the antiderivative of <i>f</i> , then $\int_{a}^{b} F(x) dx = F(b) - F(a)$ . So solve for $F(b)$ using the calculator to find the definite integral.
51. Find the derivative of $f(g(x))$	$f'(g(x)) \cdot g'(x)$
52. Given $\int_{a}^{b} f(x) dx$ , find $\int_{a}^{b} [f(x)+k] dx$	$\int_{a}^{b} [f(x)+k] dx = \int_{a}^{b} f(x) dx + \int_{a}^{b} k dx$ Make a sign chart of $f'(x)$ and determine where
53. Given a picture of $f'(x)$ , find where $f(x)$ is increasing	Make a sign chart of $f'(x)$ and determine where $f'(x)$ is positive.
54. Given $v(t)$ and $s(0)$ , find the greatest distance from the origin of a particle on $[a,b]$	Generate a sign chart of $v(t)$ to find turning points. Then integrate $v(t)$ using $s(0)$ to find the constant to find $s(t)$ . Finally, find $s($ all turning points) which will give you the distance from your starting point. Adjust for the origin.
55. Given a water tank with g gallons initially being filled at the rate of $F(t)$ gallons/min and emptied at the rate of $E(t)$ gallons/min on $[t_1, t_2]$ , find a) the amount of water in the tank at m minutes	$g + \int_{t}^{t_2} (F(t) - E(t)) dt$
56. b) the rate the water amount is changing at <i>m</i>	$\frac{d}{dt}\int_{t}^{m} (F(t) - E(t))dt = F(m) - E(m)$
57. c) the time when the water is at a minimum	F(m) - E(m) = 0, testing the endpoints as well.
58. Given a chart of x and $f(x)$ on selected values between a and b, estimate $f'(c)$ where c is between a and b.	Straddle c, using a value k greater than c and a value h less than c. so $f'(c) \approx \frac{f(k) - f(h)}{k - h}$
59. Given $\frac{dy}{dx}$ , draw a slope field	Use the given points and plug them into $\frac{dy}{dx}$ , drawing little lines with the indicated slopes at the points.
60. Find the area between curves $f(x), g(x)$ on $[a,b]$	$A = \int_{a}^{b} [f(x) - g(x)] dx$ , assuming that the <i>f</i> curve is above the <i>g</i> curve.
61. Find the volume if the area between $f(x)g(x)$ is rotated about the <i>x</i> -axis	$A = \int_{a}^{b} \left[ (f(x))^{2} - (g(x))^{2} \right] dx$ assuming that the <i>f</i> curve is above the <i>g</i> curve.

## **BC Problems**

$f(\mathbf{r})$	Use L'Henitel's Dule
62. Find $\lim_{x \to \infty} \frac{f(x)}{g(x)}$ if $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} g(x) = 0$	Use L'Hopital's Rule.
63. Find $\int_0^\infty f(x) dx$	$\lim_{h\to\infty}\int_0^h f(x)dx$
64. $\frac{dP}{dt} = \frac{k}{M}P(M-P)$ or $\frac{dP}{dt} = kP\left(1-\frac{P}{M}\right)$	Signals logistic growth. $\lim_{t \to \infty} \frac{dP}{dt} = 0 \Rightarrow M = P$
65. Find $\int \frac{dx}{x^2 + ax + b}$ where $x^2 + ax + b$ factors	Factor denominator and use Heaviside partial fraction technique.
66. The position vector of a particle moving in the plane is $r(t) = \langle x(t), y(t) \rangle$ a) Find the velocity.	$v(t) = \left\langle x'(t), y'(t) \right\rangle$
67. b) Find the acceleration.	$a(t) = \left\langle x''(t), y''(t) \right\rangle$
68. c) Find the speed.	$  v(t)   = \sqrt{[x'(t)]^2 + [y'(t)]^2}$
69. a) Given the velocity vector	$s(t) = \int x(t) dt + \int y(t) dt + C$
$v(t) = \langle x(t), y(t) \rangle$ and position at time 0, find the position vector.	Use $s(0)$ to find <i>C</i> , remembering it is a vector.
70. b) When does the particle stop?	$v(t) = 0 \rightarrow x(t) = 0 \text{ AND } y(t) = 0$
71. c) Find the slope of the tangent line to the vector at $t_1$ .	This is the acceleration vector at $t_1$ .
72. Find the area inside the polar curve $r = f(\theta)$ .	$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} \left[ f(\theta) \right]^2 d\theta$
73. Find the slope of the tangent line to the polar curve $r = f(\theta)$ .	$x = r\cos\theta, y = r\sin\theta \Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$
74. Use Euler's method to approximate $f(1.2)$ given $\frac{dy}{dx}$ , $(x_0, y_0) = (1,1)$ , and $\Delta x = 0.1$	$dy = \frac{dy}{dx} (\Delta x), y_{\text{new}} = y_{\text{old}} + dy$
75. Is the Euler's approximation an underestimate or an overestimate?	Look at sign of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in the interval. This gives you
	increasing.decreasing/concavity. Draw picture to ascertain

	answer.
76. Find $\int x^n e^{ax} dx$ where <i>a</i> , <i>n</i> are integers	Integration by parts, $\int u  dv = uv - \int v  du + C$
77. Write a series for $x^n \cos x$ where <i>n</i> is an integer	$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$ Multiply each term by $x^n$
78. Write a series for $\ln(1+x)$ centered at $x=0$ .	Find Maclaurin polynomial: $P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots + \frac{f^{(n)}(0)}{n!}x^n$
79. $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if	<i>p</i> > 1
80. If $f(x) = 2 + 6x + 18x^2 + 54x^3 +,$ find $f\left(-\frac{1}{2}\right)$	Plug in and factor. This will be a geometric series: $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$
81. Find the interval of convergence of a series.	Use a test (usually the ratio) to find the interval and then test convergence at the endpoints.
82. Let $S_4$ be the sum of the first 4 terms of an alternating series for $f(x)$ . Approximate $ f(x) - S_4 $	This is the error for the 4 <sup>th</sup> term of an alternating series which is simply the 5 <sup>th</sup> term. It will be positive since you are looking for an absolute value.
83. Suppose $f^{(n)}(x) = \frac{(n+1) n!}{2^n}$ . Write the first four terms and the general term of a series for $f(x)$ centered at $x = c$	You are being given a formula for the derivative of $f(x)$ . $f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n$
84. Given a Taylor series, find the Lagrange form of the remainder for the $n^{\text{th}}$ term where <i>n</i> is an integer at $x = c$ .	You need to determine the largest value of the 5 <sup>th</sup> derivative of <i>f</i> at some value of <i>z</i> . Usually you are told this. Then: $R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!} (x-c)^{n+1}$ $f(x) = e^x$
85. $f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$	$f(x) = e^x$
86. $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \dots$	$f(x) = \sin x$
87. $f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + \frac{(-1)^n x^{2n}}{(2n)!} + \dots$	$f(x) = \cos x$
88. Find $\int (\sin x)^m (\cos x)^n dx$ where <i>m</i> and <i>n</i> are integers	If $m$ is odd and positive, save a sine and convert everything else to cosine. The sine will be the $du$ . If n is odd and positive, save a cosine and convert everything else to sine. The cosine will be the $du$ . Otherwise use the fact that:

	$1 \cos 2r$ $1 \cos 2r$
	$\sin^2 x = \frac{1 - \cos 2x}{2}$ and $\cos^2 x = \frac{1 + \cos 2x}{2}$
89. Given $x = f(t), y = g(t)$ , find $\frac{dy}{dx}$	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$
90. Given $x = f(t), y = g(t)$ , find $\frac{d^2 y}{dx^2}$	$\frac{dt}{dt} = \frac{d}{dt} \left[ \frac{dy}{dx^2} = \frac{d}{dt} \left[ \frac{dy}{dx} \right] = \frac{\frac{d}{dt} \left[ \frac{dy}{dx} \right]}{\frac{dx}{dt}}$
91. Given $f(x)$ , find arc length on $[a,b]$	$L = \int_{a}^{b} \sqrt{1 + \left[f'(x)\right]^2} dx$
92. $x = f(t), y = g(t)$ , find arc length on	
$\begin{bmatrix} t_1, t_2 \end{bmatrix}$	$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$
93. Find horizontal tangents to a polar curve	$x = r\cos\theta, y = r\sin\theta$
$r = f(\theta)$	Find where $r\sin\theta = 0$ where $r\cos\theta \neq 0$
94. Find vertical tangents to a polar curve	$x = r\cos\theta, y = r\sin\theta$
$r = f(\theta)$	Find where $r\cos\theta = 0$ where $r\sin\theta \neq 0$
95. Find the volume when the area between $y = f(x), x = 0, y = 0$ is rotated about the <i>y</i> -axis.	Shell method: $V = 2\pi \int_{0}^{b} radius \cdot height dx$ where b is the root.
96. Given a set of points, estimate the volume under the curve using Simpson's rule on $[a,b]$ .	$A \approx \frac{b-a}{3n} \left[ y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \dots + 4y_{n-1} + y_n \right]$
97. Find the dot product: $\langle u_1, u_2 \rangle \cdot \langle v_1, v_2 \rangle$	$\langle u_1, u_2 \rangle \cdot \langle v_1, v_2 \rangle = u_1 v_1 + u_2 v_2$
98. Multiply two vectors:	You get a scalar.