Find the radius (and interval) of convergence for

1. Recall the geometric series
$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}, \quad |r| < 1$$
Let $a = 1$ and $r = x$
   (a) Write the sum of the series:

   (b) How would this be related to $f(x) = \dfrac{1}{1-x}$?

   (c) What is the interval of convergence?

   (d) Adapt the series so it is centered at -1

2. (a) Find a power Series for $f(x) = \dfrac{4}{x+2}$

Hint: write this in the form of $\dfrac{a}{1-r}$ so that once you know a and r you can write the series as $\displaystyle\sum_{n=0}^{\infty} ar^n$

(b) What is the interval of convergence?

3. (a) Find a power series for $f(x) = \dfrac{1}{x}$ centered at 1

   Hint: write this in the form of $\dfrac{a}{1-r}$— (add a 1 and subtract a 1)

   (b) What is the interval of convergence?

4. (a) Find a power series for $f(x) = \dfrac{3x-1}{x^2-1}$ centered at 0

Hint: Use partial fraction decomposition, then use the geometric series trick with both

(b) What is the interval of convergence?

5. (a) Find a power series for $f(x) = \ln x$ centered at 1

Hint: Use integration! Didn't we do a series for $\frac{1}{x}$ *before?*

$$\ln x = \int \frac{1}{x} dx + C$$

(b) What is the interval of convergence?

6. Find a power series for $f(x) = \arctan x$ centered at 0

(a) Recall

$$\frac{d}{dy}(\arctan y) = \frac{1}{1+y}$$

substitute $y = x^2$. Doesn't

$$f(x^2) = \frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n x^{2n}?$$

Now integrate to find the series for $\arctan x$

(b) It can be shown that this series for $\arctan x$ converges for $x = \pm 1$. What is the series approximation for $\arctan 1$. Use your calc's sum(seq()) function to add a 100 or so terms. Is it close to $\frac{\pi}{4}$?

Answers:
2a. $2(1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \dots)$
2b. Converges on the open interval $(-2, 2)$
3a. $1 - (x-1)^1 + (x-1)^2 - (x-1)^3 + \dots$
3b. Converges on the open interval $(0, 2)$
4a. $f(x) = \frac{3x-1}{x^2-1} = \frac{2}{x+1} + \frac{1}{x-1}$ so $1 - 3x + x^2 - 3x^3 + x^4 - 3x^5 + \dots$
4b. Converges on the open interval $(-1, 1)$
5a. To find C, let $x = 1$, so $C = 0$. $f(x) = \ln x = \frac{2}{x+1} + \frac{1}{x-1}$ so $\frac{(x-1)^1}{1} - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots$
5b. Converges on the open interval $(0, 2]$ — see how integration can change the convergence at the end points?
6a. $f(x) = \arctan x = \frac{2}{x+1} + \frac{1}{x-1}$ so $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$
6b. Converges on the open interval $(-1, 1)$
6b. Converges very slowly....