Calculus Word Problems

Name: Per.:

Often in Calculus you need to use your algebra skills to set up a word problem. Here are a few examples for you to do as homework:

1. A right circular cylinder is inscribed in a sphere of radius R (which is considered a constant). Write the equation for the surface area of the cylinder as a function of the radius of the cylinder.

2. Write the equation for the distance between the point $(0, 9)$ and the curve $x = 2y^2$ as a function of y.

3. An open box is to be made from a 3 ft by 8 ft rectangular piece of sheet metal by cutting out squares of equal size from the four corners and then bending up the sides. Write an equation for the volume of the box as a function of the length of the square piece cut from each corner.

4. A right circular cylinder is inscribed in a cone with radius 6 inches and height 10 inches.
   (a) Write an equation for the volume of the cylinder as a function of the radius of the cylinder.
   (b) Write an equation for the volume of the cylinder as a function of the height of the cylinder.

5. A wire of 12 inches can be bent into a circle, bent into a square, or cut into two pieces to make both a circle and a square.
   (a) Write an equation for the sum of the areas of the circle and the square as a function of the radius of the circle.
   (b) Write an equation for the sum of the areas of the circle and the square as a function of the length of the side of the square.

6. A rectangle has its two lower corners on the x-axis and its two upper corners on the curve $Y = 16 - x^2$. Write an equation of the area of the rectangle as a function of y.

7. Write the equation of the line (in point slope form) tangent to the curve $f(x) = (x + 2)^2$ that intersects the curve at $x = 2$ and passes through the origin. A tangent to a curve is similar to a tangent to a circle in that it intersects the curve at one point, the point of tangency.

8. Write the equation for the area of a circle as a function of its diameter.

9. Write the equation for the circumference of a circle as a function of its area.

10. Write the equation for the surface area of a sphere as a function of its volume.

11. A 17 ft ladder is leaning against a wall. If the bottom of the ladder is x feet from the wall and the top of the ladder is y feet above the ground, write an equation for the horizontal distance the ladder is from the bottom of the wall as a function of the vertical distance the ladder is above the ground.

12.) Wheat is poured through a chute and falls in a conical pile whose radius is always half the altitude of the pile.
   (a) Write an equation for the volume of the pile as a function of the altitude of the pile.
   (b) Write an equation for the volume of the pile as a function of the radius of the pile.

13. A container with a square base, vertical sides, and open top is to be made from 1000 ft^2 of material.
   (a) Write an equation for the volume of the container as a function of the length of a side of the base.
   (b) Write an equation for the volume of the container as a function of the length of the vertical side.

14. An aircraft is climbing at a 30° angle to the horizontal. Write the equation of the height of the airplane above the ground as a function of its distance from the airport on the ground.

15. A rectangle is to have a perimeter of 100 m. Write an equation for the area of the rectangle as a function of one of its sides.

16. A plank is to be used to reach over a fence 8 ft high to support a wall 1 ft behind the fence. Write an equation for the length of the plank as a function of the angle the plank makes with the ground.

17. A right circular cone is inscribed in a sphere of radius R (which is a constant). Write the equation for the volume of the cone as a function of the radius of the cone.

18. Sketch the functions, shade the region enclosed by the functions, and label all points of intersection on the boundary of the functions $y = x^2$ and $y = x + 6$

19. Sketch the functions, shade the region enclosed by the functions, and label all points of intersection on the boundary of the functions $y = x$, $y = 4x$ and $y = -x + 2$.

20. Sketch the functions, shade the region enclosed by the functions, and label all points of intersection on the boundary of the functions $y = \cos 2x$, $y = 0$, and $x = \dfrac{\pi}{2}$.

21. Sketch the functions, shade the region enclosed by the functions, and label all points of intersection on the boundary of the functions $x = y^2$ and $x = y + 3$.

22. A camera is positioned 400 meters from the base of a missile launching pad and is tracking a 100 meters long missile as it is launched vertically upward. The zoom of the camera continually adjusts itself so that the top and bottom of the missile fill the entire screen. Write an equation for the angle (of the camera) that subtends the missile as a function of the height the bottom of the missile is above the launch pad.

23. A cup is in the shape of a truncated cone with a radius of 4 cm at the top and 2 cm at the bottom and a height of 6 cm. Water is being poured into the cup such that the height of the water in the cup is changing. Write an equation for volume of the water in the cup as a function of its height.