

Examples:

A. How many ways are there to arrange the letters “CORRECTLY”?

Solution : Since there are 2 C’s, and 2 R’s: $\binom{9}{2\ 1\ 2\ 1\ 1\ 1\ 1} = \frac{9!}{2!1!2!1!1!1!} = 90,720$

B. What is the probability that a random arrangement of CCRROECTLY will spell “CORRECTLY”?

Solution 1: (from above) $\frac{1}{90,720}$ Solution 2: $\left(\frac{2}{9}\right)\left(\frac{1}{8}\right)\left(\frac{2}{7}\right)\left(\frac{1}{6}\right)\left(\frac{1}{5}\right)\left(\frac{1}{4}\right)\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)\left(\frac{1}{1}\right)$

C. Three people will be interviewed. How many possible ways are there to order the interviews?

Solution: A Permutation, ${}_3P_3 = 3! = 3*2*1 = 6$

D. If five boys and five girls sit in a row in a random order, what is the probability that no two children of the same sex sit together?

Solution: $\frac{\text{ways of sitting boy - girl}}{\text{ways of sitting 10 people}} = \frac{2*5!*5!}{10!}$ (boys sit in position 1,3,5,7,9 and girls sit in 2,4,6,8,10 or vice versa)

E. Your locker has 12 books. If you randomly pick out two, how many different pair are possible?

Solution: $\binom{12}{2} = {}_{12}C_2 = \frac{12!}{2!10!} = \frac{12*11}{2} = 66$

Exercises

1. Show that the number of distinguishable permutations of MISSISSIPPI is 34,650
2. A dancing contest has 3 Americans, 2 Mexicans, 3 Russians, and 3 Italians. If the contest result lists only the nationality of the dancers, show the number of possible outcomes are 92,400.
3. How many ways can you rearrange the letters AFRICA so that it still begins and ends with an “A”?
4. Five boys and five girls are randomly assigned a seat. Show that the probability that the boys are seated together is 0.0079.
5. Six fair dice are tossed. Show the probability that at least two of them show the same face is 0.985.
6. If you have 9 friends, and need 3 for a bridge game, show that there are ___ combinations.
7. If there are 150 seniors, show there are ___ possible ways a President, VP, Treasurer and Secretary could be selected.

More Probability

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \text{ and } B) = P(A)P(B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Examples:

G. There are 100 dice in a box. 35 are plastic, the rest are wood. there are 40 red and 60 blue. 10 of the wood dice are red. One is selected at random.

	RED	BLUE	TOTAL
PLASTIC	30	5	35
WOOD	10	55	65
TOTAL	40	60	100

Probability of plastic = P(Plastic)=30/100=.3, Probability of Wood = 1-P(Plastic)=1-.3=.7

Probability of a Blue = P(Blue)=.6, P(Red)=.4 Probability of Red and Wood = .1

$$P(Plastic|Blue) = \frac{5}{60} = \frac{1}{12} \quad P(Blue|Plastic) = 0 \quad P(Wood|Blue) = \frac{P(Wood \cap Blue)}{P(Blue)} = \frac{.55}{.6} = .9167$$

H. A certain cancer is found in 1 in 5000. A certain test is 92% accurate for those with the disease, and 99.8% accurate for those who do not. If you get a positive result, what is the probability you actually have the cancer?

	Pos. test	Neg. test
Cancer	.92	Type I
No Cancer	Type II	.998

$$P(Can|Pos) = \frac{P(Can)P(Pos|Can)}{P(Can)P(Pos|Can) + P(\overline{Can})P(Pos|\overline{Can})} = \text{around } .084$$

Exercises

8. There are 10 students, 3 are boys, 6 are redheads and 3 girls are not redheads.
 - a. Make a Venn Diagram
 - b. What is the probability of picking a red headed boy?
 - c. What is the probability of picking a girl, given the student is a redhead?
 - d. What is the probability of picking a redhead, given the student is a girl?
 - e. What is the probability of picking a boy or a redhead?