Partial Fraction Decompostion Worksheet

Name:

Steps:

1. If the numerator has a degree higher than the denominator (i.e., improper), divide so $\dfrac{N}{D} = (\mathit{polynomial}) + \dfrac{N_1}{D}$
2. With the remaining fraction, factor the denominator D into terms that are either linear factors of the form $(px+q)^m$ and quadratic factors of the form $(ax^2+bx+c)^n$, where ax^2+bx+c is irreducible.
3. For each linear factor Find $A_1, A_2, \ldots, A_m$ so that $\dfrac{N_1}{D} = \dfrac{A_1}{(px+q)} + \dfrac{A_2}{(px+q)^2} + \cdots + \dfrac{A_m}{(px+q)^m}$
4. For each quadratic, do the same so that $\dfrac{N_1}{D} = \dfrac{B_1x+C_1}{(ax^2+bx+c)} + \dfrac{B_2x+C_2}{(ax^2+bx+c)^2x+C_n} + \cdots + \dfrac{B_n}{(ax^2+bx+c)^n}$

Example:

$$\frac{x^3+x^2}{(x^2+4)^2}$$

The numerator is degree 3, and the denominator is degree 4, so we don't have to divide since this is a proper fraction. Now we make up an A, B, C and D so that

$$\frac{x^3+x^2}{(x^2+4)^2} = \frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+4)^2}$$

We clear the fractions and get:

$$x^3+x^2 = (Ax+B)(x^2+4) + Cx + D$$

Collect the terms and we have:

$$x^3+x^2 = Ax^3 + Bx^2 + (4A+C)x + D + 4B$$

By looking at the coefficients, we have four equations for our four unknowns:

$$\begin{aligned} A &= 1 \\ B &= 1 \\ 4A + C &= 0 \\ D + 4B &= 0 \end{aligned}$$

So $A = 1$, $B = 1$, $C = -4$ and $D = -4$, so we have finally:

$$\frac{x^3+x^2}{(x^2+4)^2} = \frac{x+1}{(x^2+4)} + \frac{-4x-4}{(x^2+4)^2}$$

Try These:

1. $\dfrac{3x+1}{2(x+1)}$
2. $\dfrac{7x+3}{(x+1)(x-1)}$
3. $\dfrac{x^2+2x+3}{(x^2+4)^2}$
4. $\dfrac{7x+3}{x^3-2x^2-3x}$
5. $\dfrac{x^2}{x^3-4x^2+5x-2}$
6. $\dfrac{x^3}{(x^2+16)^3}$
7. $\dfrac{2x+3}{x^4-9x^2}$

Answers (**Answers**) to *reflect* upon

1. $\frac{3}{2} - \frac{1}{x+1}$ 2. $\frac{2}{x+1} + \frac{5}{x-1}$

3. $\frac{1}{x^2+4} + \frac{2x-1}{(x^2+4)^2}$ 4. $\frac{-1}{x} + \frac{2}{x-3} + \frac{-1}{x+1}$

5. $\frac{4}{x-2} + \frac{-3}{x-1} + \frac{-1}{(x-1)^2}$ 6. $\frac{x}{(x^2+16)^2} + \frac{-16x}{(x^2+16)^3}$

7. $\frac{-2}{9(x)} + \frac{-1}{3(x^2)} + \frac{1}{6(x-3)} + \frac{1}{18(x+3)}$