

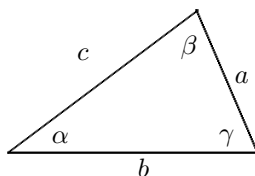
1. Right Triangles

$$a^2 + b^2 = c^2$$

$$a = c \sin \alpha = c \cos \beta = b \tan \alpha = \frac{b}{\tan \beta}$$

$$\beta = \tan^{-1} \frac{b}{a} = \cos^{-1} \frac{a}{c} = \sin^{-1} \frac{b}{c}$$

2. "Law of Cosines"



(a) Side-Angle-Side (SAS)

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

Double the product of the sides you know, then subtract that from the sum of the squares of the two sides, and then take the square-root of the result.

(b) Side-Side-Side (SSS)

$$\alpha = \cos^{-1} \left( \frac{b^2 + c^2 - a^2}{2bc} \right)$$

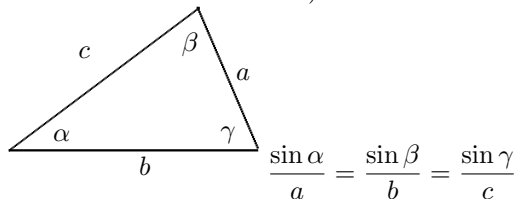
$$\beta = \cos^{-1} \left( \frac{a^2 + c^2 - b^2}{2ac} \right)$$

$$\gamma = \cos^{-1} \left( \frac{a^2 + b^2 - c^2}{2ab} \right)$$

Answers:

1.  $\alpha = 14.48^\circ$ ,  $\beta = 75.52^\circ$ ,  $\gamma = 100.00^\circ$   
 2.  $\alpha = 80^\circ$ ,  $\beta = 27.8^\circ$ ,  $\gamma = 92.2^\circ$   
 3.  $\alpha = 28.2^\circ$ ,  $\beta = 111.17^\circ$ ,  $\gamma = 60.6^\circ$   
 4.  $\alpha = 32.8^\circ$ ,  $\beta = 149.0^\circ$ ,  $\gamma = 17.2^\circ$   
 5.  $\alpha = 24.0^\circ$ ,  $\beta = 157.8^\circ$ ,  $\gamma = 18.2^\circ$

3. If you know 3 of the 6 arranged the other ways (Section 9.2: "Law of Sines")



- (a) Any 2 angles and the side in between the known angles (ASA) or 2 angles and a side (AAS/ASA/SAA): Only one solution possible.
- (b) 2 Angles, and NOT the side in between (ASS, SSA)...Zero, one or two solutions possible. Method: Find a  $\beta_1$ , then consider the possible  $\beta_2 = 180 - \beta_1$ 
  - i. No solutions if  $\sin \beta > 1$  or  $\sin \beta < -1$
  - ii. One solution if  $180 - \alpha - \beta_2 \leq 0$ ...use  $\beta_1$  as the unique solution
  - iii. Two solutions otherwise (you need to consider  $\beta_1$  to get  $c_1$  and  $\gamma_1$  and then consider  $\beta_2$  to find  $c_2$  and  $\gamma_2$ )

