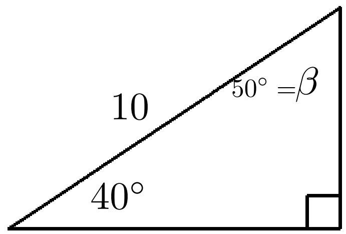
Right Triangles



$a = 10\sin(40^\circ) \approx 6.43$

$b = 10\cos(40^\circ) \approx 7.66 \quad = 10\sin(50^\circ)$

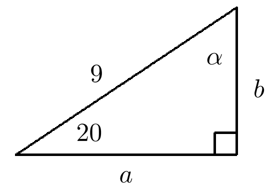
Proof:

$10 \quad \cos(40^\circ) = \frac{b}{\cancel{10}} \ \cancel{10}$

Also...

9.1(p. 633)

9.

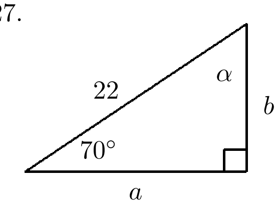


$\cos 20^\circ = \frac{a}{9}$ $\quad$ $\sin 20^\circ = \frac{b}{9}$

so $a = 9\cos 20^\circ$ ≈ 8.457 $\quad$ so $b = 9\sin 20^\circ$ ≈ 3.078

$\alpha = 90 - 20 = 70^\circ$

27.



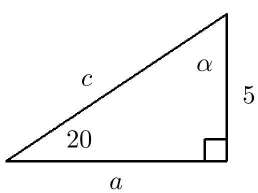
$\cos 70^\circ = \frac{a}{22}$ $\quad$ $\sin 70^\circ = \frac{b}{22}$

so $a = 22\cos 70^\circ$ ≈ 7.524 $\quad$ so $b = 22\sin 70^\circ$ ≈ 20.673

$\alpha = 90 - 70 = 20^\circ$

9.1 (p. 633)

1.

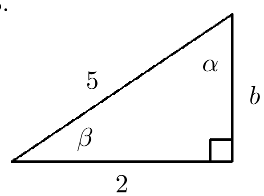


$\tan 20^\circ = \frac{5}{a}$ $\quad$ $\sin 20^\circ = \frac{5}{c}$

So $a = \frac{5}{\tan 20^\circ}$ ≈ 13.74 $\quad$ So $c = \frac{5}{\sin 20^\circ}$ ≈ 14.62

$\alpha = 90 - 20 = 70^\circ$

13.



$\cos\beta = \frac{2}{5}$ $\quad$ $\sin\alpha = \frac{2}{5}$

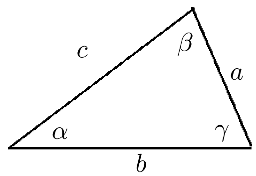
So $\beta = \cos^{-1}\left(\frac{2}{5}\right)$ $\approx 66.42^\circ$ $\quad$ So $\alpha = \sin^{-1}\left(\frac{2}{5}\right)$ $\approx 23.58^\circ$

$b = \sqrt{5^2 - 2^2} = \sqrt{21}$

What about Triangles that AREN'T Right Triangles?

Law of Sines

If you know two angles and a side

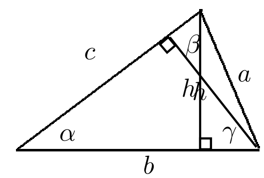


$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

or for that matter...

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Proof



$h = a\sin\beta$

$h = b\sin\alpha$

$$\frac{\cancel{a}\sin\beta}{\cancel{a}b} = \frac{\cancel{b}\sin\alpha}{\cancel{b}a}$$

$h = c\sin\alpha$

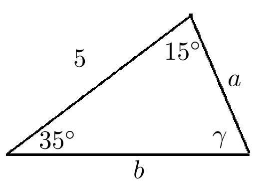
$h = a\sin\gamma$

$$\frac{\cancel{c}\sin\alpha}{\cancel{c}a} = \frac{\cancel{a}\sin\gamma}{\cancel{a}c}$$

$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

ASA Case (1 solution)



$\gamma = 180 - 35 - 15 = 130$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

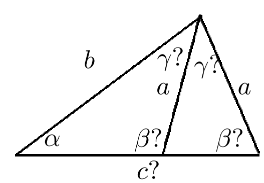
$$\frac{\sin 35}{a} = \frac{\sin 15}{b} = \frac{\sin\gamma}{5}$$

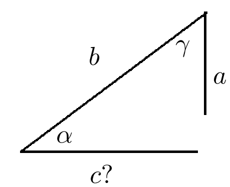
$$\frac{\sin 35}{a} = \frac{\sin 15}{b} = \frac{\sin 130}{5}$$

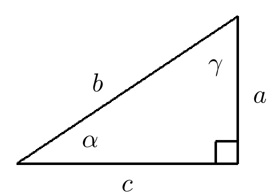
$$\frac{\sin 35}{a} = \frac{\sin 130}{5} \qquad \frac{\sin 15}{b} = \frac{\sin 130}{5}$$

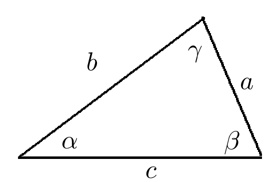
$$a = \frac{5\sin 35}{\sin 130} \approx 3.74 \qquad b = \frac{5\sin 15}{\sin 130} \approx 1.69$$

SAA/ASS Ambiguity







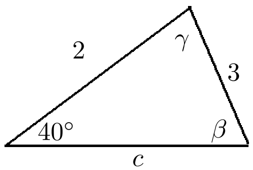


See page 640 for details...

Don't Panic... it sorts itself out...

The Solution?

After finding β, consider $180-\beta$



Let $\beta_1 = 25.4°$ and let's consider the supplement, $\beta_2 = 180 - 25.4 = 154.6°$

We know $\alpha = 40$,
so if we consider $\alpha + \beta_2 = 40 + 154.6 = 194°$!!!!

So $\beta_1 = 25.4°$ is the unique solution!

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

$$\frac{\sin 40}{3} = \frac{\sin\beta}{2}$$

$$\sin\beta = \frac{2\sin 40}{3}$$

$$\beta = \sin^{-1}\left(\frac{2\sin 40}{3}\right) \approx 25.4°$$

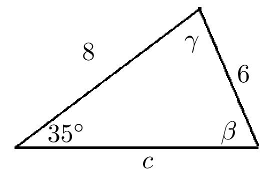
$$\gamma = 180 - 40 - 24.4 = 114.6$$

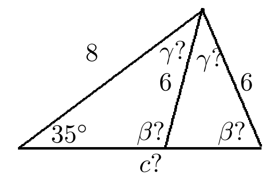
$$\frac{\sin\alpha}{a} = \frac{\sin\gamma}{c}$$

$$\frac{\sin 40}{3} = \frac{\sin 114.6}{c}$$

$$c = \frac{3\sin 114.6}{\sin 40} \approx 4.24$$

Could Both Beta's Work?





$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

$$\frac{\sin 35°}{6} = \frac{\sin\beta}{8}$$

$$\sin\beta = \frac{8\sin 35°}{6}$$

$$\beta = \sin^{-1}\left(\frac{8\sin 35°}{6}\right) \approx 49.9°$$

$$\beta_2 = 180 - 49.9 = 130.1°$$

$$\alpha + \beta_2 = 35 + 130.1 = 165.1°(< 180)$$

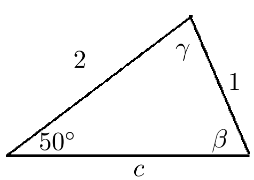
If β_1=49.9, $\gamma_1 = 180 - 35 - 49.9 = 95.1$

$$c_1 = \frac{6\sin 95.1}{\sin 35} \approx 10.42$$

If $\beta_2 = 130.1°$, $\gamma_2 = 180 - 35 - 130.1 = 14.9°$

$$c_2 = \frac{6\sin 14.9}{\sin 35} \approx 2.69$$

Could neither work?



$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

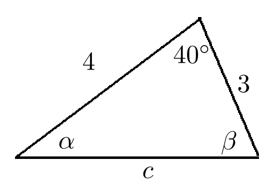
$$\frac{\sin 50}{1} = \frac{\sin\beta}{2}$$

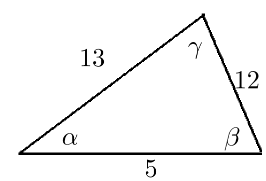
$$\sin\beta = 2\sin 50 = 1.532088886???$$

But doesn't sine stay between 1 and -1?????

$\sin^{-1}(1.532088886) =$ cppXbmk_gl

What about SAS & SSS?





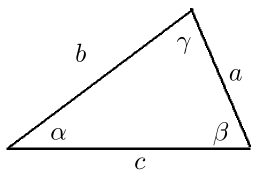
$$\frac{\sin\alpha}{3} = \frac{\sin\beta}{4} = \frac{\sin 40}{c}$$

$$\frac{\sin\alpha}{12} = \frac{\sin\beta}{13} = \frac{\sin\gamma}{5}$$

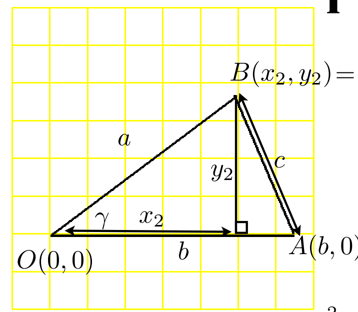
Law of Sines won't work!

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$



Proof



$B(x_2, y_2) = (a\cos\gamma, a\sin\gamma)$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$c = \sqrt{(b - a\cos\gamma)^2 + (0 - a\sin\gamma)^2}$$

$$c^2 = (b - a\cos\gamma)^2 + (0 - a\sin\gamma)^2$$

$$c^2 = (b^2 - 2ab\cos\gamma + a^2\cos^2\gamma) + (a^2\sin^2\gamma)$$

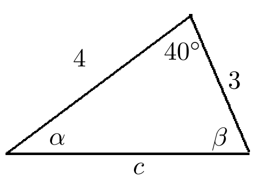
$$c^2 = b^2 - 2ab\cos\gamma + (a^2\cos^2\gamma + a^2\sin^2\gamma)$$

$$c^2 = b^2 - 2ab\cos\gamma + a^2(\cos^2\gamma + \sin^2\gamma)$$

$$c^2 = b^2 - 2ab\cos\gamma + a^2(1)$$

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos\gamma}$$

SAS
page 651#9



$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$c^2 = 3^2 + 4^2 - 2(3)(4)\cos 40°$$

$$c^2 = 9 + 16 - 24\cos 40°$$

$$c^2 = 25 - 24\cos 40°$$

$$c = \sqrt{25 - 24\cos 40°} \approx 2.57$$

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

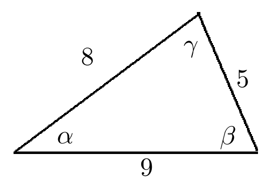
$$2bc\cos\alpha = b^2 + c^2 - a^2$$

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\alpha = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right) = \cos^{-1}\left(\frac{4^2 + (2.57)^2 - 3^2}{2(4)(2.57)}\right) \approx 48.6°$$

$$\beta = 180 - \alpha - \gamma = 180 - 48.6 - 40 = 91.4°$$

SSS
page 651#21



$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\alpha = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$\alpha = \cos^{-1}\left(\frac{8^2 + 9^2 - 5^2}{2(8)(9)}\right) \approx 33.56°$$

$$\beta = \cos^{-1}\left(\frac{5^2 + 9^2 - 8^2}{2(5)(9)}\right) \approx 62.18°$$

$$\gamma = \cos^{-1}\left(\frac{5^2 + 8^2 - 9^2}{2(5)(8)}\right) \approx 84.26°$$