Hypberbola WS

Name:

Examples

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Center: (h, k) Asymptotes are the diagonals of the box made if you go up and down b from k and left and right a from h. Vertices are at $(h \pm a, k)$ Foci are $(h \pm c, k)$ where $c = \sqrt{a^2 + b^2}$

1. Graph the equation

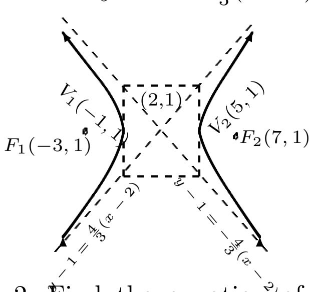
$$\frac{(x-2)^2}{9} - \frac{(y-1)}{16} = 1$$

This is centered at (2,1), $a = 3, b = 4$, $c = \sqrt{9 + 16} = 5$
The slopes of the asymptotes are $\pm\frac{4}{3}$. Use the point-slope form of a line with the center:

$$y - k = \pm m(x - h)$$

we get $y - 1 = \frac{4}{3}(x - 2)$ aka $y = \frac{4}{3}x - \frac{5}{3}$

and $y - 1 = -\frac{4}{3}(x - 2)$ aka $y = -\frac{4}{3}x + \frac{11}{3}$



2. Find the equation of the hyperbola with Foci $F_1(13, 3)$ and $F_2(-7, 3)$ and vertex $V_2(11, 3)$

Solution The center is half way from F_1 and F_2: $\frac{13+(-7)}{2} = 3$, so the center is (3,3). That means the vertex (11,3) is 8 away from the center, so $a = 8$, $c = 10$ and $b = \sqrt{100 - 64} = 6$. The equation is then:

$$\frac{(x-3)^2}{64} - \frac{(y-3)^2}{36} = 1$$

The vertical version works similarly, but the x^2 term is negative:

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Center: (h, k) Asymptotes are the diagonals of the box made if you go up and down a from k and left and right b from h. Vertices are at $(h, k \pm a)$ Foci are $(h, k \pm c)$ where $c = \sqrt{a^2 + b^2}$

Now you try

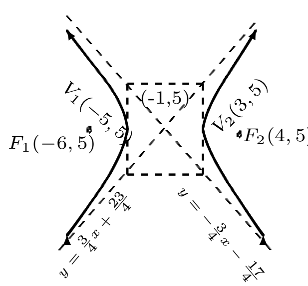
1. Graph

$$\frac{(x-1)^2}{144} - \frac{(y-3)^2}{25} = 1$$

2. Graph

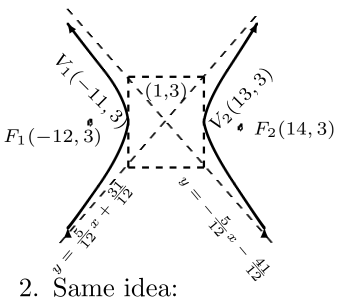
$$\frac{x^2}{4} - \frac{(y-1)^2}{9} = 1$$

3. Find the equation of

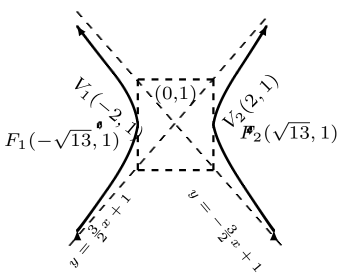


Solutions

1. Here the center is (1,3), $a = 12, b = 5$ and $c = \sqrt{144+25} = 13$ so $F_1(-12,3)$. Draw a box going up and down 5 from (1,3) and left and right 12 from (1,3). The Asymptotes are $y-3 = \frac{5}{12}(x-1)$ or $y = \frac{5}{12}x + \frac{31}{12}$ and $y - 3 = -\frac{5}{12}(x-1)$ or $y = -\frac{5}{12}x - \frac{41}{12}$



2. Same idea:



3.
$$\frac{(x+1)^2}{16} - \frac{(y-5)^2}{9} = 1$$