The Difference Quotient, or "rate of change" (otherwise known as slope!) is

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

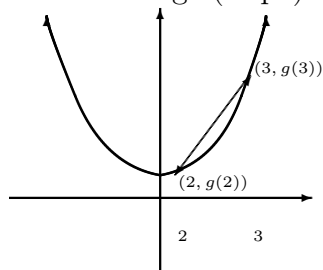So the "average rate of change" of a function from x to x+h is

$$\frac{\Delta y}{\Delta x} = \frac{f(x + h) - f(x)}{h}$$

Here are some examples:

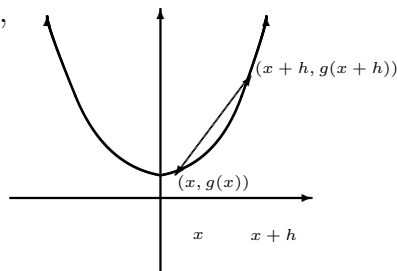**A.** Consider the difference quotient of $g(x) = x^2 + 2$ from 2 to 3:

$$\frac{g(3) - g(2)}{3 - 2} = \frac{(3^2 + 2) - (2^2 + 2)}{1}$$
$$= 11 - 6$$
$$= 5$$

This means the slope of the secant line is 5. In other words, from 2 to 3, the average rate of change (slope) is 5.

B. What is the difference quotient for $g(x) = x^2 + 2$ between 1 and x?

$$\frac{g(x) - g(1)}{x - 1} = \frac{(x^2 + 2) - (1^2 + 2)}{x - 1}$$
$$= \frac{x^2 + 2 - 3}{x - 1}$$
$$= \frac{x^2 - 1}{x - 1}$$
$$= \frac{(x - 1)(x + 1)}{x - 1}$$
$$= x + 1$$

C. What is the diffence quotient for $g(x) = x^2 + 2$ between x and $x + h$?

$$\frac{g(x + h) - g(x)}{(x + h) - x} = \frac{g(x + h) - g(x)}{h}$$
$$= \frac{((x + h)^2 + 2) - (x^2 + 2)}{h}$$
$$= \frac{(x^2 + 2xh + h^2 + 2) - x^2 - 2}{h}$$
$$= \frac{2xh + h^2}{h}$$
$$= 2x + h$$

So here we have a formula. The answer from part C can confirm the answer from part A. In part A, let $x = 2$ and $h = 1$ (so $x + h = 3$). Then the average rate of change is

$$2x + h = 2(2) + 1 = 5$$

If the function is a line, the slope is always the same. If the function is some other curve, however, it will change. In that case the slope depends on which x, and how much h is added to x.
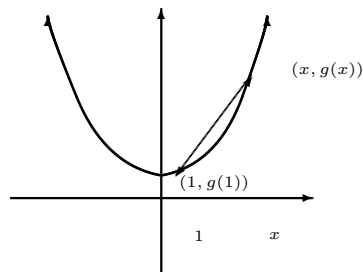
1. If $f(x) = 5x - 3$, what is the average rate of change of f from 2 to 3?

2. What is the average rate of change of f between 1 and x?

3. What is the average rate of change of f between x and $x + h$?

4. If $g(x) = 3x^2 - 5x$, what is the average rate of change of g from 1 to 2?

5. What is the average rate of change of g between 1 and x?

6. What is the average rate of change of g between x and $x + h$?

7. If $p(x) = \dfrac{1}{x - 1}$, what is the average rate of change of p from 2 to 3?

8. What is the average rate of change of p between 2 and x?

9. What is the average rate of change of p between x and $x + h$?

Answers:

(1) 5 (2) 5 (3) 5 (4) 4 (5) 3x − 2

(6) $6x + 3h - 5$ (7) $-\frac{1}{2}$ (8) $\frac{1}{1-x}$ (9) $-\frac{1}{(x+h-1)(x-1)}$

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