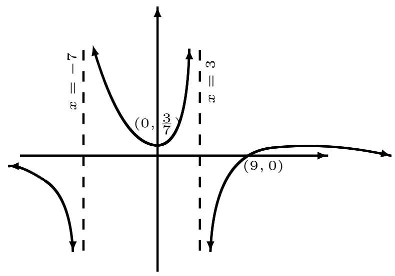
Asymptotes

Name: Block: Seat:

1. Vertical (look for division by zero)
   Ex 1. $f(x) = \dfrac{x-9}{x^2+4x-21} = \dfrac{x-9}{(x+7)(x-3)}$

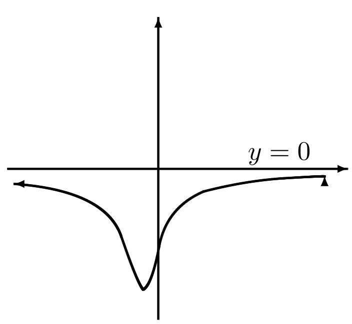
   So there will be two vertical asymptotes:

   $x = -7$ and $x = 3$.



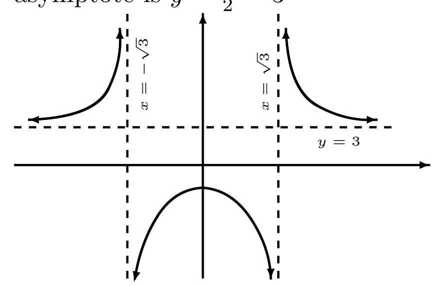
2. Horizontal (What happens to $f(x)$ as $x \to \pm\infty$)
   Horizontal asymptotes may exist only when the degree of the numerator is less than or equal to the degree of the denominator. Consider only the terms with the highest degree of x in the top and bottom of the ratio:

   (a) **Top Degree < Bottom Degree or "Proper fraction"** Horizontal asymptote $y = 0$, as in our first example.
   + Ex 2a. $f(x) = \dfrac{x-12}{4x^2+x+1}$
   So there is a horizonal asymptote $y = 0$ (the x=axis).



   (b) **Top Degree= Bottom Degree** Horizontal asymptote $y = \dfrac{a_n}{b_n}$

   Ex 2b. $f(x) = \dfrac{6x^2+5}{2x^2-6}$ Both are degree 2, and $a_2 = 6$ and $b_2 = 2$ so the horizontal asymptote is $y = \frac{6}{2} = 3$



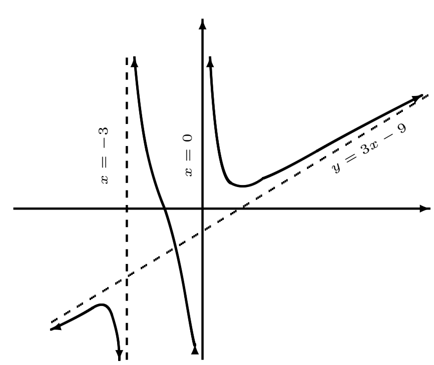
3. Oblique (look for a numerator with a higher degree than the denominator)
   Use long division to find the "whole part" of the mixed fraction. This will be the equation of the oblique asymptote.
   Ex 3. $f(x) = \dfrac{3x^3+4}{x^2+3x}$

$$
\begin{array}{r|rrrr}
 & & & 3x & -9 \\
\hline
x^2+3x & 3x^3 & 0x^2 & +0x & +4 \\
 & -(3x^3 & 9x^2) & & \\
\hline
 & & -9x^2 & +0x & \\
 & & -(-9x^2 & -27x) & \\
\hline
 & & & -27x & +4
\end{array}
$$

   So the function

$$
\begin{aligned}
f(x) = \frac{3x^3+4}{x^2+3x} &= 3x - 9 + \frac{-27x+4}{x^2+3x} \\
&= 3x - 9 + \frac{-27x+4}{x(x+3)}
\end{aligned}
$$

   is inbetween the oblique asymptote $y = 3x - 9$ and the vertical asymptote $x = -3$



1. $f(x) = \dfrac{x-2}{x^2-3x-4}$

2. $f(x) = \dfrac{x-1}{x^2-2x}$

3. $f(x) = \dfrac{3x^2}{2x^2+1}$

4. $f(x) = \dfrac{x^2}{x+1}$

5. $f(x) = \dfrac{x^2-2x+1}{x}$

Answers

(1)Vertical: $x = 4$ and $x = -1$ Horiz: $y = 0$

(2)Vertical: $x = 0$ and $x = 2$ Horiz: $y = 0$

(3) No vertical and Horiz: $y = \frac{3}{2}$

(4) $f(x) = x - 1 + \frac{1}{x+1}$ Vertical $x = -1$ Oblique: $y = x - 1$

(5) $f(x) = x - 2 + \frac{1}{x}$ Vertical $x = 0$ Oblique: $y = x - 2$