Recall that...

If a, b, and x are positive numbers and $\dfrac{a}{x} = \dfrac{x}{b}$, then x is called the **geometric mean** between a and b. In other words, geometric mean of a and b is equal to $\sqrt{ab}$.

Theorem 8-1

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangle formed are similar to the original triangle and to each other.
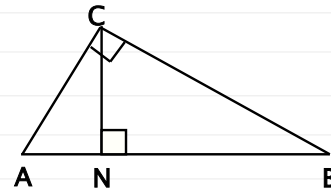
Given: $\triangle ABC$ with rt. $\angle ACB$; altitude segment CN

Prove: $\triangle ACB \sim \triangle ANC \sim \triangle CNB$

1

Corollary 1

When the altitude is drawn to the hypotenuse of a right triangle, the length of the altitude is the geometric mean between the segments of the hypotenuse.
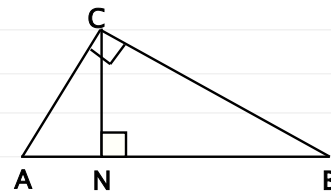
Given: $\triangle ABC$ with rt. $\angle ACB$; altitude segment CN
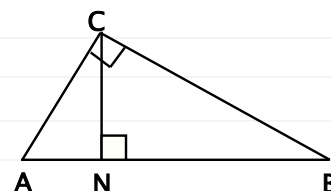
Prove: $AN \cdot BN = CN^2$

Corollary 2

When the altitude is drawn to the hypotenuse of a right triangle, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse that is adjacent to that leg.

Given: $\triangle ABC$ with rt. $\angle ACB$; altitude segment CN

Prove: (1) $AB \cdot AN = AC^2$; (2) $AB \cdot BN = BC^2$

2

on your desk

8.1

8.2

8.3

8.4

8.5

8.6

8.7

Example

1. If JR=16 and EJ=9, find HJ, RE, RH, and HE.
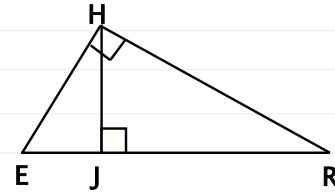
3. If JR=4 and JH=2, find JE, RE, RH, and HE.

4. Simplify each expression.
   (a) $\sqrt{75}$ (b) $\sqrt{\dfrac{2}{5}}$

   (c) $2\sqrt{48}$ (d) $\sqrt{16}\cdot\sqrt{4}$

on your desk

8.1

8.2

8.3

8.4

8.5

8.6

8.7

Practice

Simplify.

1. $3\sqrt{50}$ 4. $\dfrac{8}{2\sqrt{2}}$

2. $\sqrt{7}\cdot\sqrt{14}$ 5. $\sqrt{45}\cdot\sqrt{5}$

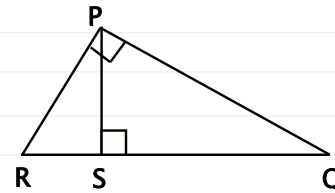3. $\dfrac{12}{\sqrt{3}}$ 6. $\sqrt{\dfrac{3}{4}}$

7. If RS=2 and SQ=8, find PS.

8. If RP=10 and RS=5, find SQ.

9. If RS=4 and PS=6, find SQ.

on your desk

8.1

8.2

8.3

8.4

8.5

8.6

8.7

Theorem 8.2 (Pythagorean Theorem)

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

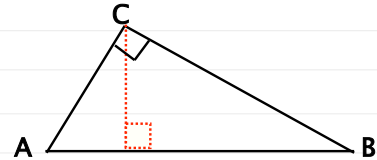Given: $\triangle$ABC with rt. $\angle$ACB

Prove:

Statements	Reasons

5

---

on your desk

8.1

8.2

8.3

8.4

8.5

8.6

8.7

Example

Find the value of y.

1.

5

y

10

3.

5

y

y−1

4.

2x

x

$5\sqrt{3}$

6

on your desk

Practice

Find the value of each variable.

8.1
8.2
8.3
8.4

1.

x

2

3

4. Find the length of a diagonal of a rectangle with length 8 and width 4.

8.5
8.6
8.7

3.

y

4

6

4.

4

x

x
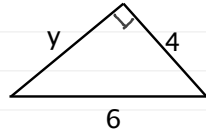
7

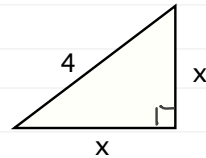on your desk

Theorem 8.3 (The Converse of the Pythagorean Theorem)

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

8.1
8.2
8.3
8.4

Pythagorean Triples Family

3, 4, 5   5, 12, 13   8, 15, 17   7, 24, 25

8.5   6, 8, 10

8.6   9, 12, 15

8.7

8

How do you tell if a triangle is right, obtuse, or acute?

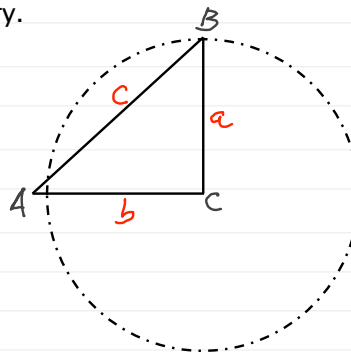Hint 1: Most eloquent theorem in Geometry.

m∠C is right: $c^2 = a^2 + b^2$

m∠C is obtuse: $c^2 > a^2 + b^2$

m∠C is acute: $c^2 < a^2 + b^2$

NOTE: c is the longest side.

Example

Determine if the following set of number form a triangle, then decide if the triangle is right, obtuse, or acute.

(a) 3,4,5   (b) 3,4,6   (c) 5,6,7

Example

The sides of a triangle have the lengths given. Is the triangle acute, right, or obtuse?

1. 20, 21, 29

2. 20, 21, 30

3. 20, 21, 28

4. 5, 6, 8

on your desk

8.1

8.2

8.3

8.4

8.5

8.6

8.7

Practice

Classify each triangle with sides of the given lengths as acute, right, or obtuse.
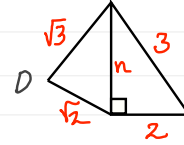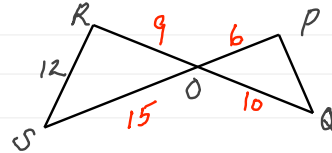
1. 5, 12, 14

2. 6, 7, 8

3. $\sqrt{3}, \sqrt{4}, \sqrt{5}$

4. 1, 3, $\sqrt{10}$

5. 9, 40, 41

6. Explain why $\angle D$ must be a right angle.

7. Explain why $\angle P$ must be a right angle.

on your desk

8.1

8.2

8.3

8.4

8.5

8.6

8.7

Theorem 8.6 (45–45–90 Theorem)

In a 45°–45°–90° triangle, the hypotenuse is _____ times as long as a leg.
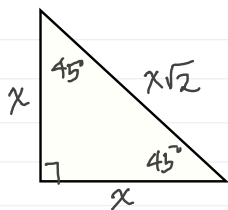

Theorem 8.7 (45–45–90 Theorem)

In a 30°–60°–90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is _____ times as long as the shorter leg.
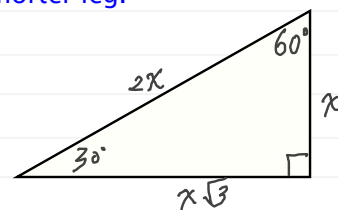
on your desk

Example

Find the value of y.

8.1

8.2

8.3

8.4

1. Find the value of x.

8.5

8.6

3. Find the value of x and y.

8.7

.

---

on your desk

Practice

Complete

8.1

1. If r=6, t=_____.

8.2

8.3

2. If s=$2\sqrt{5}$, t=_____.

8.4

3. If t= $\sqrt{2}$, r=_____.

8.5

8.6

4. If t=10, s=_____.

8.7
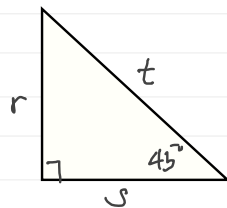
.

on your desk

Practice

Complete

8.1

8.2

8.3

8.4

8.5

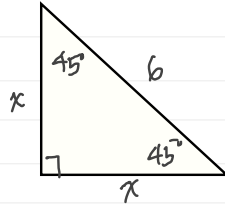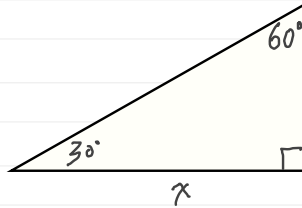8.6

8.7

5. If q=8, p=_____ and n=_____.

6. If n=20, q=_____ and p=_____.

7. If p= $4\sqrt{3}$, q=_____ and n=_____.

8. If p=9, q=_____, and n=_____.

9. A diagonal of a square has length 6.
    What is the perimeter of the square?

15

.

on your desk

Practice

Find the value of x.

8.1

8.2

8.3

8.4

10.

8.5

8.6

8.7

11.

12. In regular hexagon ABCDEF, AB=8. Find AD and AC.

16

.

tangent of $\angle A = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$

opposite
leg

A

adjacent leg

Example

Find tan A, tan B, m∠A, and m∠B.

1. C   7   A
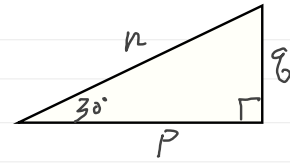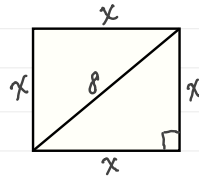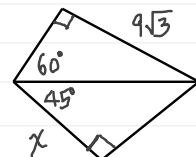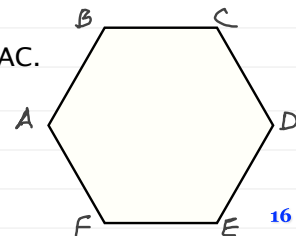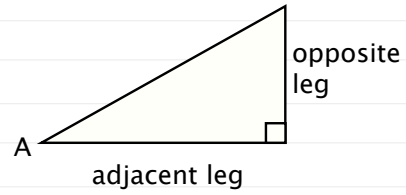
  2

  B

2.

A

17

C   8   B

17
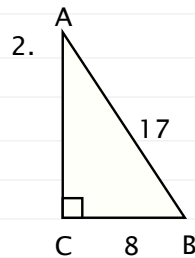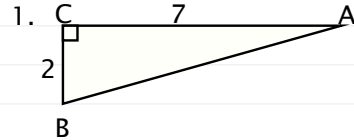
EX Find the sine, cosine, and tangent of 53°

θ	sin(•)	cos(•)	tan(•)		•	sin(•)	cos(•)	tan(•)
0	0.000	1.000	0.000		45	0.707	0.707	1.000
1	0.017	1.000	0.017		46	0.719	0.695	1.036
2	0.035	0.999	0.035		47	0.731	0.682	1.072
3	0.052	0.999	0.052		48	0.743	0.669	1.111
4	0.070	0.998	0.070		49	0.755	0.656	1.150
5	0.087	0.996	0.087		50	0.766	0.643	1.192
6	0.105	0.995	0.105		51	0.777	0.629	1.235
7	0.122	0.993	0.123		52	0.788	0.616	1.280
8	0.139	0.990	0.141		53	0.799	0.602	1.327
9	0.156	0.988	0.158		54	0.809	0.588	1.376
10	0.174	0.985	0.176		55	0.819	0.574	1.428
11	0.191	0.982	0.194		56	0.829	0.559	1.483
12	0.208	0.978	0.213		57	0.839	0.545	1.540
13	0.225	0.974	0.231		58	0.848	0.530	1.600
14	0.242	0.970	0.249		59	0.857	0.515	1.664
15	0.259	0.966	0.268		60	0.866	0.500	1.732

TABLE OF TRIGONOMETRIC RATIOS

on your desk

3. Find the value of x to the nearest tenth.

20

24°

x
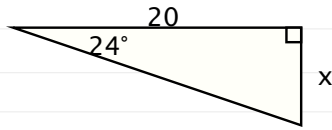
8.1

8.2

8.3

8.4

8.5

8.6

8.7

4. The grade of a road is the ratio of its rise to its run and is usually given as a decimal or percent. Find the angle that the road makes with the horizontal if its grade is 4% (4/100).

x°

rise

run

on your desk

Problem

Find x to the nearest tenth.

Find y° correct to the nearest degree.

8.1

8.2

8.3

8.4
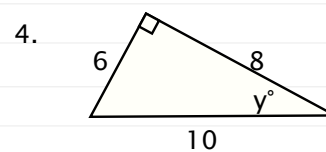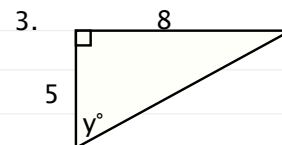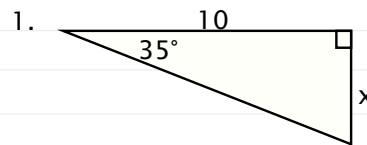
1.

10

35°

x

3.

8

5

y°

8.5

8.6

8.7

2.

30

21°

x

4.

6

8

y°

10

on your desk

8.1

8.2

8.3

8.4

8.5

8.6

8.7

$$\text{sine of } \angle A = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

$$\text{cosine of } \angle A = \frac{\text{adjacent leg}}{\text{hypotenuse}}$$

opposite leg

A
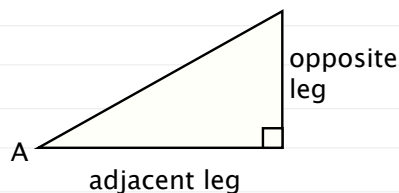
adjacent leg

SOH–CAH–TOA

"sine is opposite over hypotenuse – cosine is adjacent over hypotenuse – tangent is opposite over adjacent"

21

EX Angle A is an acute angle. Use the table to approximate the measure of angle A.

a.   Sin A = 0.12

b.   Cos A = 0.68

TABLE OF TRIGONOMETRIC RATIOS

θ	sin(•)	cos(•)	tan(•)	•	sin(•)	cos(•)	tan(•)
0	0.000	1.000	0.000	45	0.707	0.707	1.000
1	0.017	1.000	0.017	46	0.719	0.695	1.036
2	0.035	0.999	0.035	47	0.731	0.682	1.072
3	0.052	0.999	0.052	48	0.743	0.669	1.111
4	0.070	0.998	0.070	49	0.755	0.656	1.150
5	0.087	0.996	0.087	50	0.766	0.643	1.192
6	0.105	0.995	0.105	51	0.777	0.629	1.235
7	0.122	0.993	0.123	52	0.788	0.616	1.280
8	0.139	0.990	0.141	53	0.799	0.602	1.327
9	0.156	0.988	0.158	54	0.809	0.588	1.376
10	0.174	0.985	0.176	55	0.819	0.574	1.428
11	0.191	0.982	0.194	56	0.829	0.559	1.483
12	0.208	0.978	0.213	57	0.839	0.545	1.540
13	0.225	0.974	0.231	58	0.848	0.530	1.600
14	0.242	0.970	0.249	59	0.857	0.515	1.664
15	0.259	0.966	0.268	60	0.866	0.500	1.732
16	0.276	0.961	0.287	61	0.875	0.485	1.804
17	0.292	0.956	0.306	62	0.883	0.469	1.881
18	0.309	0.951	0.325	63	0.891	0.454	1.963
19	0.326	0.946	0.344	64	0.899	0.438	2.050
20	0.342	0.940	0.364	65	0.906	0.423	2.145

on your desk

8.1

8.2

8.3

8.4

8.5

8.6

8.7

Example

1. find the values of x and y to the nearest integer.

y

28°

x   1000
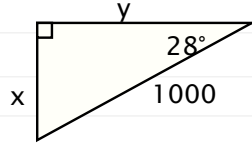
2. Find x° correct to the nearest degree.

x°   30

18

3.

A

5   5

B   4   C
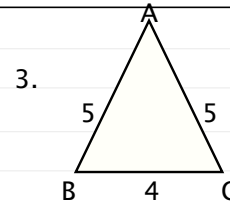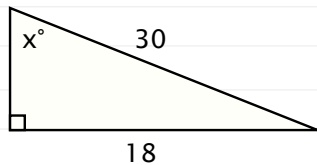
a) Find the measure of the three angles of △ABC.

b) Find the lengths of the three altitude of △ABC.

23

on your desk

8.1

8.2

8.3

8.4

8.5

8.6

8.7

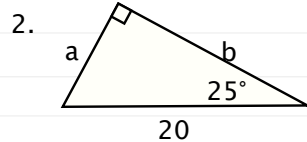## Problem

Find the value of each variable.

1.
y
56°
12   x

2.
a   b
25°
20

3.
x°
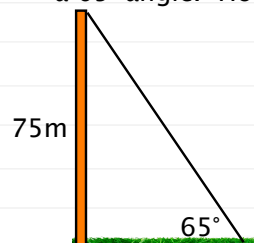5   10
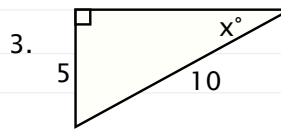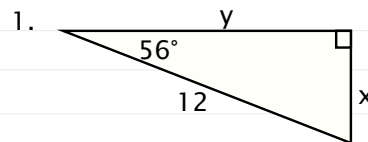
4. A guy wire is attached to the top of a 75 m tower and meets the ground at a 65° angle. How long is the wire?

75m

65°

24

on your desk

8.1

8.2

8.3

8.4

8.5

8.6

8.7

horizontal

angle of depression 2°

angle of elevation 2°

horizontal

If the top of the lighthouse is 25 m above sea level, the distance x between the boat and the base of the lighthouse can be found in these two ways:

---

on your desk

8.1

8.2

8.3

8.4

8.5

8.6

8.7

<u>Example</u>

1. Draw a diagram showing a person who is 1.5 m tall standing 20m from the base of a building. Also show that the top of the building with an angle of elevation of 58°. Find the height of the building.

2. A driveway has a 15% grade.
   a) What is the angle of elevation of the driveway?

   b) If the driveway is 12m long, about how much does it rise?

Example

3. A toboggan travels from point A at the top of the hill to the point B at the bottom. Because the steepness of the hill varies, the angle of depression from A to B is only an approximate measure of the hill's steepness. We can, however, think of this angle of depression as representing the average steepness.

   a) If the toboggan travels 130m from A to B and the vertical descent AC is 50m, what the approximate angle of depression?

   b) Why is your answer approximate?

A

C   B

27