Euler's Method

This method is based on the local linearity concept we covered earlier: If we zoom in enough on a differentiable function f at a point (a,b) the function looks linear and can be approximated by the linear equation $y = f(a) + f'(a)(x-a)$. We will use the local linearization at a point to calculate another point nearby. Then, we will use the new point and its local linearization to calculate another point. We continue this iterative process until we reach a desired stopping point. We will increase x by a fixed amount and calculate the new y-values according to the local linearization.

1. Use Euler's Method starting at $(0,1)$ with $\Delta x = 0.2$ to estimate $y(1)$ given that $y' = y$. Round to 4 decimal places. **Note.** Slope in this problem is given by y:

Step	x	Approximate y-value	$\Delta y = (\text{Slope})\Delta x$	Notes:
0	0	1	$1 \cdot 0.2 = 0.2$	Add Δy to y to get new y and increase to next x by Δx each step.
1	0.2	1.2	$1.2 \cdot 0.2 = 0.24$	
2	0.4	1.44	$1.44 \cdot 0.2 = 0.288$	
3		1.728		
4	0.8			
5	1.0			

Use your calculator or computer to draw the slope field of the differential equation. Find the actual solution to the differential equation using the initial point $(0,1)$. Is your approximation to $y(1)$ using Euler's Method an overestimate or an underestimate? Why?

2. Use Euler's Method starting at $(1,2)$ with $\Delta x = 0.2$ to estimate $y(2)$ if $y' = \frac{y}{x}$. Round to 4 decimal places. Note: Slope in this problem is given by _______________.

Step	x	Approximate y-value	$\Delta y = (\text{Slope})\Delta x$
0	1	2	$\left(\frac{2}{1}\right) \cdot 0.2 = 0.4$
1	1.2		

Use your calculator or computer to draw the slope field of the differential equation. Find the actual solution to the differential equation using the initial point $(1,2)$. Is your approximation to $y(2)$ using Euler's Method an overestimate or an underestimate? Why?

3. Use Euler's Method starting at $(0,3)$ with $\Delta x = 0.5$ to estimate $y(2)$ given that $y' = 2xy + 2y$. Use your calculator or computer to draw the slope field of the differential equation. Find the actual solution to the differential equation using the initial point $(0,3)$. Is your approximation to $y(2)$ using Euler's Method an overestimate or an underestimate? Why?

4. Repeat the previous problem except use $\Delta x = 0.25$. What can you say about your new approximation for $y(2)$?