Ultra Violet VooDoo 1 Name: Block: Seat:

$$\int u\,dv = uv - \int v\,du$$

Determine the antiderivative of each of the following. Confirm your answer by differentiation. Which is u? Let LIATE (or LIPET) be your guide!

1. LIATE (LIPET)
   (a) Logs
   (b) Inverse Trig
   (c) Algebraic (Polynomials)
   (d) Trig (Exp)
   (e) Exponential (Trig)

2. $$\int u\,dv = uv - \int v\,du$$

$$\int x \sin x\; dx$$

$u = x$	$v = -\cos x$
$du = dx$	$dv = \sin\, dx$

$= -x\cos x - \int \cos x\,dx$

$= -x\cos x + \sin x + C$

3. $uv - \int v\,du$

$$\int y \ln y\; dy$$

$u = \ln y$	$v = \frac{1}{2}y^2$
$du = \frac{1}{y}dy$	$dv = y\,dy$

$\frac{1}{2}y^2 \ln y - \int \frac{1}{2}y^2 \frac{1}{y}\,dy$

$\frac{1}{2}y^2 \ln y - \frac{1}{2}\int y\,dy$

$\frac{1}{2}y^2 \ln y - \frac{1}{2}\frac{y^2}{2} + C$

$\frac{y^2}{2}\left[\ln y - \frac{1}{2}\right] + C$

4.

$u = z^2$	$v = \sin z$
$du = 2z\,dz$	$dv = \cos z\,dz$

$$\int z^2 \cos z\; dz$$

$uv - \int v\,du$

$z^2 \sin z - \int 2z \sin z\,dz$

$z^2 \sin z - 2\int z \sin z\,dz$

$u = z$	$v = -\cos z$
$du = dz$	$dv = \sin z\,dz$

$z^2 \sin z - 2\left[-z\cos z - \int -\cos z\,dz\right]$

$z^2 \sin z + 2z\cos z + 2\int \cos z\,dz$

$z^2 \sin z + 2z\cos z + 2\sin z + C$

5.

$uv - \int u\,dv$ Att. $u_2 dv_2 - \int v_2\,du_2$ $u_3 dv_3 - \int v_3 du$

$$\int x^3 e^{-2x}\; dx$$

	(D) u	(I) dv
+	x^3	e^{-2x}
−	$3x^2$	$-\frac{1}{2}e^{-2x}$
+	$6x$	$\frac{1}{4}e^{-2x}$
−	6	$-\frac{1}{8}e^{-2x}$
+	0	$\frac{1}{16}e^{-2x}$

$e^{-2x}\left[-\frac{1}{2}x^3 - \frac{3}{4}x^2 + \frac{3}{4}x - \frac{3}{8}\right] + C$

6.

$$\int_0^{\pi/2} x^2 \sin 2x \, dx$$

	D	I
+	x^2	$\sin 2x$
−	$2x$	$\frac{-1}{2}\cos 2x$
+	2	$\frac{-1}{4}\sin 2x$
−	0	$\frac{1}{8}\cos 2x$

$$\int_0^{\pi/2} x^2 \sin 2x \, dx =$$

$$\left. -\frac{x^2}{2}\cos 2x + \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x \right|_0^{\pi/2}$$

$$\left(-\frac{\pi^2}{8}\cos\pi + \frac{\pi}{4}\sin\pi + \frac{1}{4}\cos\pi\right) - \left(0 + 0 + \frac{1}{4}\cos 0\right)$$

$$+\frac{\pi^2}{8} + 0 - \frac{1}{4} - \left(+\frac{1}{4}\right)$$

$$\text{or} \quad \frac{\pi^2}{8} - \frac{1}{2} = \frac{\pi^2 - 4}{8}$$

7.

$$\int_{-3}^{2} e^{-2x} \sin 2x \, dx$$

$u = e^{-2x}$	$v = -\frac{1}{2}\cos 2x$
$du = -2e^{-2x}\,dx$	$dv = \sin 2x \, dx$

$$\left. -\frac{1}{2}e^{-2x}\cos 2x \right|_{-3}^{2} - \int_{-3}^{2}\left(-\frac{1}{\cancel{2}}\cos 2x\right)\left(-\cancel{2}e^{-2x}\right)dx$$

$$\left. -\frac{1}{2}e^{-2x}\cos 2x \right|_{-3}^{2} - \int_{-3}^{2} e^{-2x}\cos 2x \, dx$$

$u = e^{-2x}$	$v = \frac{1}{2}\sin 2x$
$du = -2e^{-2x}\,dx$	$dv = \cos 2x \, dx$

$$\left. -\frac{1}{2}e^{-2x}\cos 2x \right|_{-3}^{2} - \left. \frac{1}{2}e^{-2x}\sin 2x \right|_{-3}^{2} + \int\left(\frac{1}{2}\sin 2x\right)\left(-2e^{-2x}\right)dx$$

$$\int_{-3}^{2} e^{-2x}\sin 2x \, dx = \left. -\frac{1}{2}e^{-2x}\cos 2x - \frac{1}{2}e^{-2x}\sin 2x \right|_{-3}^{2} - \int e^{-2x}\sin 2x \, dx$$

$$2\int_{-3}^{2} e^{-2x}\sin 2x \, dx = \left. -\frac{1}{2}e^{-2x}(\cos 2x + \sin 2x) \right|_{-3}^{2}$$

$$\int_{-3}^{2} e^{-2x}\sin 2x \, dx = \left. -\frac{1}{4}e^{-2x}(\cos 2x + \sin 2x) \right|_{-3}^{2} = -\frac{1}{4}\left[\frac{1}{e^4}(\cos 4 + \sin 4) - e^{6}(\cos 6 + \sin 6)\right]$$

$$= \frac{1}{4}\left[e^{6}(\cos 6 + \sin 6) - e^{-4}(\cos 4 + \sin 4)\right]$$

Answers:

2. $\sin(x) - x\cos(x) + C$
3. $\frac{1}{2}y^2 \ln y - \frac{1}{4}y^2 + C$
4. $z^2 \sin z - 2\sin z + 2z\cos z + C$
5. $-\frac{1}{8}e^{-2x}(4x^3 + 6x^2 + 6x + 3) + C$
6. $\frac{x}{2}\sin(2x) - \frac{1}{4}(2x^2 - 1)\cos(2x)|_0^{\pi/2}$
7. $-\frac{1}{4}e^{-2x}(\sin(2x) + \cos(2x))|_{-3}^{2}$

$$\int_{-3}^{2} e^{-2x}\sin 2x\,dx$$

$u = e^{-2x}$	$v = -\frac{1}{2}\cos 2x$
$du = -2e^{-2x}\,dx$	$dv = \sin 2x\,dx$

$$-\frac{1}{2}e^{-2x}\cos 2x\Big|_{-3}^{2} - \int_{-3}^{2}\left(-\frac{1}{\cancel{2}}\cos 2x\right)\left(-\cancel{2}e^{-2x}\right)dx$$

$$-\frac{1}{2}e^{-2x}\cos 2x\Big|_{-3}^{2} - \int_{-3}^{2} e^{-2x}\cos 2x\,dx$$

$u = e^{-2x}$	$v = \frac{1}{2}\sin 2x$
$du = -2e^{-2x}\,dx$	$dv = \cos 2x\,dx$

$$-\frac{1}{2}e^{-2x}\cos 2x\Big|_{-3}^{2} - \frac{1}{2}e^{-2x}\sin 2x\Big|_{-3}^{2} + \int\left(\frac{1}{2}\sin 2x\right)\left(-2e^{-2x}\right)dx$$

$$\int_{-3}^{2} e^{-2x}\sin 2x\,dx = -\frac{1}{2}e^{-2x}\cos 2x - \frac{1}{2}e^{-2x}\sin 2x\Big|_{-3}^{2} - \int e^{-2x}\sin 2x\,dx$$

$$2\int_{-3}^{2} e^{-2x}\sin 2x\,dx = -\frac{1}{2}e^{-2x}(\cos 2x - \sin 2x)\Big|_{-3}^{2}$$

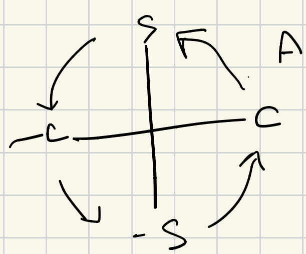
$$\int_{-3}^{2} e^{-2x}\sin 2x\,dx = -\frac{1}{4}e^{-2x}(\cos 2x - \sin 2x)\Big|_{-3}^{2} = -\frac{1}{4}\left[\frac{1}{e^4}(\cos 4 + \sin 4) - e^{6}(\cos 6 - \sin 6)\right]$$

$$= \frac{1}{4}\left[e^{6}(\cos 6 - \sin 6) - e^{-4}(\cos 4 + \sin 4)\right]$$

Table Method of $\int e^{-2x}\sin 2x\,dx$

	D	I
+	$\sin 2x$	e^{-2x}
−	$2\cos 2x$	$-\frac{1}{2}e^{-2x}$
+	$-4\sin 2x$	$\frac{1}{4}e^{-2x}$
−	$-8\cos 2x$	$-\frac{1}{8}e^{-2x}$
+	$16\sin 2x$	$\frac{1}{16}e^{-2x}$

	D	I
+	e^{-2x}	$\sin 2x$
−	$-2e^{-2x}$	$-\frac{1}{2}\cos 2x$
+	$4e^{-2x}$	$-\frac{1}{4}\sin 2x$
−	$-8e^{-2x}$	$\frac{1}{8}\cos 2x$
+	$16e^{-2x}$	$\frac{1}{16}\sin 2x$



$u = e^{-2x}$ $\quad v = -\frac{1}{2}\cos 2x$