Euler's Method

Euler's Method is used to generate numerical approximations for solutions to differential equations. It is necessary to know an initial point and a rate of change (the derivative) for the function. Euler's Method uses locally linear approximations at successive steps to estimate the solutions/

Required information: (x_0, y_0) an initial point

$\frac{dy}{dx}$ the derivative

dx the differential or increment of *x*

Starting at the initial point (x_0, y_0), the next point is found by using the formulas

$x_1 = x_0 + \Delta x$ and $y_1 = y_0 + \Delta y.$

But we don't know the exact value of $\Delta y,$ so we can approximate it by dy when the increment Δx is very small. So,

$x_1 = x_0 + dx$ and $y_1 = y_0 + f'(x_0, y_0) * dx.$

Subsequent points are found by using the general formulas

$x_{n+1} = x_n + dx$ and $y_{n+1} = y_n + f'(x_n, y_n) * dx.$

Example 1. Given $\frac{dy}{dx} = 2x + y$ with $y(0) = 1$ and $dx = 0.1,$ find the first two approximations y_1 and y_2 using Euler's Method.

It is helpful to organize your information in a table

Initial x	Initial y	$\Delta x = dx$	$\Delta y \approx dy = f'(x,y) * dx$ $dy = (2x + y)dx$	New x = Initial $x + dx$	New y= Init $y + dy$
0	1				