Calc BC Arclength WS

Name: Block: Seat:

The distance formula is basically the pythagorean theorem, since you can imagine connecting the points to make a segment, and the segment becomes the hypotenuse of a right triangle if you imagine the legs are the rise (Δy) and the run (Δx)

$$\sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$$

Arclength is the distance along a curve. You can approximate it by adding up all the segments made by chopping it up into the segments made from $(x_i, f(x_i))$.

$$\lim_{\Delta x \to 0} \sum_{i=0}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$$

Take a look at
http://geogebra.org/m/547883
to see $f(x) = \sqrt{x}$ from 0 to 1.

Since the first derivative of the curve is $\frac{\Delta y}{\Delta x}$, lets get that into the act by replacing Δy with $\frac{\Delta y}{\Delta x} * \Delta x$:

$$\lim_{\Delta x \to 0} \sum_{i=0}^{n} \sqrt{(\Delta x_i)^2 + \left(\frac{\Delta y_i}{\Delta x}\right)^2 (\Delta x)^2}$$

Now we factor out the common $(\Delta x)^2$:

$$\lim_{\Delta x \to 0} \sum_{i=0}^{n} \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x}\right)^2} \Delta x$$

By the continuity of f and the Intermediate Value Theorem, we realize we have our arc length from $f(a)$ to $f(b)$:

$$\boxed{\int_a^b \sqrt{1 + [f'(x)]^2} dx}$$

Symmetrically we can argue if the x coordinate can be described is a continuous function g where $x = g(y)$, we have

$$\boxed{\int_a^b \sqrt{1 + [g'(y)]^2} dy}$$

1. Find the arc length from 0 to 1 of $\sqrt{x}$

2. Find the arc length from (a, b) to (c, d) on the graph of $f(x)$ where $f'(x) = \frac{d-b}{c-a}$

3. Find the arc length of the graph of $(y-1)^3 = x^2$ on the interval $[0, 8]$
*Hint:*find x in terms of y

Rotation around an Axis

Remember (Section 7.2) finding the volume of a solid by using the disk method? The idea is to use the area of a circle ($A = \pi r^2$), where the volume of a disk is the circular area times the width

$$V = \pi R^2 w$$

so when the radius of the disk changes as a function $R(x)$, and the width w is the Δx width so that

$$\Delta V_i = \pi[R(x_i)]^2 * \Delta x$$

so we add them all together to get the volume formula:

$$V = \pi \int_a^b [R(x)]^2 dx$$

The same kind of idea can be used with the circumference formula ($C = 2\pi r$) to get the surface area of a object made by rotating around an axis we multiply the circumference by the arc length L:

$$A = 2\pi R * L$$

$$\Delta A_i = 2\pi r_i * \Delta L_i$$

and since $L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$ we can arrive at the formula:

$$\boxed{A = 2\pi \int_a^b R(x)\sqrt{1 + [f'(x)]^2} dx}$$

(full description in Section 7.4, pp.480-481)
`http://geogebra.org/m/1342821`
has a red/blue 3d view

4. Find the area of the surface formed by revolving the graph of $f(x) = x^3$ on the interval [0,1] about the x-axis. *Hint:* $r(x)$ (the distance between the x-axis and graph is $f(x)$

5. Find the area of the surface formed by revolving the graph of $f(x) = x^2$ on the interval $[0, \sqrt{2}]$ about the y-axis. *Hint:* In this case the distance between the graph and the y-axis is $r(x) = x$