Find the radius (and interval) of convergence for

1.
$$\sum_{n=0}^{\infty} n!x^n$$

2.
$$\sum_{n=0}^{\infty} 3(x-2)^n$$

3.
$$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

4.

$$\sum_{n=0}^{\infty} \frac{x^n}{n}$$

5.

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x+1)^n}{2^n}$$

6. Consider

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n}$$

Using Thm 9.21 (p. 664):

(a) Find the interval of convergence of $\int f(x)dx$

(b) Find the interval of convergence of $f(x)$

(c) Find the interval of convergence of $f'(x)$

Answers:

1. By Ratio test diverges for $|x| > 0$, converges only at the center 0, so radius is $R = 0$
2. By Ratio test converges for $|x-2| < 1$, diverges if $|x-2| > 1$, interval is $(1, 3)$ so radius is $R = 1$
3. By Ratio test converges for all x, interval is $(-\infty, \infty)$ so radius is $R = \infty$
4. By Ratio test converges for all $x \in (-1, 1)$, so radius is $R = 1$,
check end points: diverges for $x = 1$ (harmonic), converges for $x = -1$ (alt. harmonic) so interval is $[-1, 1)$
5. By Ratio test converges if $\left|\frac{x+1}{2}\right| < 1$ so all $x \in (-3, 1)$, so radius is $R = 2$,
check end points: diverges for $x = -3$, diverges for $x = 1$ so interval is $(-3, 1)$
6. for all radius is $R = 1$
6a. interval is $[-1, 1]$
6b. interval is $[-1, 1)$
6c. interval is $(-1, 1)$